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Autoequivalences of tensor categories attached to quantum groups at roots of 1. (English)

Zbl 07065397
Kac, Victor G. (ed.) et al., Lie groups, geometry, and representation theory. A tribute to the life and work of Bertram Kostant. Cham: Birkhäuser (ISBN 978-3-030-02190-0/hbk; 978-3-030-02191-7/ebook). Progress in Mathematics 326, 109-136 (2018).

Let k be an algebraically closed field of characteristic zero, G a simple algebraic group over k, $\mathfrak{g}$ its Lie algebra, q a root of unity of odd order coprime to 3 if G is of type G_2 and coprime to the determinant of the Cartan matrix of G, and $\mathfrak{u}_q(\mathfrak{g})$ Lusztig's small quantum group attached to $\mathfrak{g}$ [*G. Lusztig*, J. Am. Math. Soc. 3, No. 1, 257–296 (1990; Zbl 0695.16006)]. Then $\mathfrak{u}_q(\mathfrak{g})$ is a quasitriangular Hopf algebra so that the category $\mathsf{Rep}\,\mathfrak{u}_q(\mathfrak{g})$ of its finite-dimensional representations is a finite braided tensor category [*P. Etingof* et al., Tensor categories. Providence, RI: American Mathematical Society (AMS) (2015; Zbl 1365.18001)].

One of the principal goals in this paper is to compute the Picard group $\mathsf{Pic}(\mathsf{Rep}\,\mathfrak{u}_q(\mathfrak{g}))$ of this category, namely, the group of equivalence classes of invertible $\mathsf{Rep}\,\mathfrak{u}_q(\mathfrak{g})$-module categories. It is well known that $\mathsf{Pic}(\mathsf{Rep}\,\mathfrak{u}_q(\mathfrak{g}))$ is isomorphic to the group $\mathsf{Aut}^{\mathrm{bt}}(\mathsf{Rep}\,\mathfrak{u}_q(\mathfrak{g}))$ of braided autoequivalences of $\mathsf{Rep}\,\mathfrak{u}_q(\mathfrak{g})$ [*A. Davydov* and *D. Nikshych*, Algebra Number Theory 7, No. 6, 1365–1403 (2013; Zbl 1284.18015); *P. Etingof* et al., Quantum Topol. 1, No. 3, 209–273 (2010; Zbl 1214.18007)]. It is shown under some restriction on q that $\mathsf{Aut}^{\mathrm{bt}}(\mathsf{Rep}\,\mathfrak{u}_q(\mathfrak{g}))$ is isomorphic to $\mathsf{Aut}(\mathfrak{g})$ of automorphisms of $\mathfrak{g}$, namely,

$$\mathsf{Aut}^{\mathrm{bt}}(\mathsf{Rep}\,\mathfrak{u}_q(\mathfrak{g})) = \Gamma \rhd\!< G^{\mathrm{ad}}$$

where G^{ad} is the adjoint group of G and $\Gamma = \Gamma_{\mathfrak{g}}$ is the automorphism group of the Dynkin diagram of $\mathfrak{g}$. It is also shown that $\mathsf{Rep}\,\mathfrak{u}_q(\mathfrak{g})$ has only two braidings, namely, the standard one and its inverse, any tensor autoequivalence of $\mathsf{Rep}\,\mathfrak{u}_q(\mathfrak{g})$ being necessarily braided. Therefore, the group of tensor autoequivalences of $\mathsf{Rep}\,\mathfrak{u}_q(\mathfrak{g})$ is isomorphic to $\Gamma \rhd\!< G^{\mathrm{ad}}$, which generalizes the result in [*J. Bichon*, Glasg. Math. J. 58, No. 3, 727–738 (2016; Zbl 1377.16025)] dealing with the case that $\mathfrak{g} = \mathfrak{sl}_2$.

In consideration of the braided tensor category $\mathcal{O}_q(G)-\mathsf{comod}$ of finite-dimensional comodules over the function algebra $\mathcal{O}_q(G)$, which is the G-equivariantization of $\mathsf{Rep}\,\mathfrak{u}_q(\mathfrak{g})$, it is shown that every braided autoequivalence of $\mathcal{O}_q(G)-\mathsf{comod}$ comes from a Dynkin diagram automorphism if l is sufficiently large, a similar result being established in the non-braided case, which generalizes a result in [*S. Neshveyev* and *L. Tuset*, Adv. Math. 227, No. 1, 146–169 (2011; Zbl 1220.46046); Int. Math. Res. Not. 2012, No. 15, 3498–3508 (2012; Zbl 1256.17008)] dealing with the case that q is not a root of unity. This paper deals with the classical groups SL_N, Sp_N and SO_N when $l > N$.

This paper introduces the notion of a finitely dominated tensor category as a gadget. It is shown that the category of comodules over a finitely presented Hopf algebra is finitely dominated, it being established that tensor autoequivalences of a finitely dominated category preserving a tensor generator form an algebraic group.

For the entire collection see [Zbl 1412.22001].

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MSC:

18M20 Fusion categories, modular tensor categories, modular functors
17B37 Quantum groups (quantized enveloping algebras) and related deformations
81R50 Quantum groups and related algebraic methods applied to problems in quantum theory
20G42 Quantum groups (quantized function algebras) and their representations

Keywords:

tensor category; quantum group; autoequivalence; braiding; quantum Frobenius

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