Baez, John C.; Courser, Kenny
Structured cospans. (English) Zbl 07281441
Theory Appl. Categ. 35, 1771-1822 (2020).

This paper aims to introduce *structured cospans* as a way to study networks with inputs and outputs and then to show that they are advantageous over *decorated cospans*.

A synopsis of the paper goes as follows. Given a functor $L : \mathsf{A} \to \mathsf{X}$, §2 defines the notion of a *structured cospan*, which is of the form

$$\begin{array}{ccccc} & & x & & \\ & \overset{i}{\nearrow} & & \overset{o}{\nwarrow} & \\ L(a) & & & & L(b) \end{array}$$

where a and b are called the *input* and *output*, respectively, while x is called the *apex* with the morphisms i and o named the *legs* of the cospan. It is shown in Theorem 2.3 that for any functor $L : \mathsf{A} \to \mathsf{X}$, when X has pushouts, there is a double category ${}_L\mathbb{C}\boldsymbol{sp}(\mathsf{X})$ in which

- objects are objects of A,
- vertical 1-morphisms are morphisms of A,
- horizontal 1-cells are structured cospans,
- 2-morphisms are commutative diagrams $\begin{array}{ccccc} L(a) & \overset{i}{\rightarrow} & x & \overset{o}{\leftarrow} & L(b) \\ L(\alpha)\downarrow & & f\ \downarrow & & \downarrow L(\beta) \\ L(a') & \underset{i'}{\rightarrow} & x' & \underset{o'}{\leftarrow} & L(b') \end{array}$

It is shown in Corollary 2.4 that for any functor $L : \mathsf{A} \to \mathsf{X}$, when X has pushouts, there is a bicategory ${}_L\boldsymbol{Csp}(\mathsf{X})$ in which

- objects are objects of A,
- 1-morphisms are morphisms of A,
- 2-morphisms are commutative diagrams in X of the form

$$\begin{array}{ccccc} & & x & & \\ & \nearrow & & \nwarrow & \\ L(a) & & \downarrow & & L(b) \\ & \searrow & & \swarrow & \\ & & x' & & \end{array}$$

It is shown in Corollary 2.5 that for any functor $L : \mathsf{A} \to \mathsf{X}$, when X has pushouts, there is a category ${}_L\mathsf{Csp}(\mathsf{X})$ in which

- objects are objects of A,
- morphisms are isomorphism classes of structured cospans, where two structured cospans $L(a) \to x \leftarrow L(b)$ and $L(a) \to y \leftarrow L(b)$ are said to be *isomorphic* if there is an isomorphism $f : x \to y$ such that the diagram

$$\begin{array}{ccccc} & & x & & \\ & \nearrow & & \nwarrow & \\ L(a) & & f\ \downarrow & & L(b) \\ & \searrow & & \swarrow & \\ & & y & & \end{array}$$

  commutes.

§3 gives simple conditions under which the double category ${}_L\mathbb{C}\boldsymbol{sp}(\mathsf{X})$ as well as the bicategory ${}_L\boldsymbol{Csp}(\mathsf{X})$ and the category ${}_L\mathsf{Csp}(\mathsf{X})$ becomes symmetric monoidal. The monoidal structure is based on the formation of

$$L(a+a') \xrightarrow{\cong} L(a)+L(a') \xrightarrow{i+i'} x+x' \xleftarrow{o+o'} L(b)+L(b') \xleftarrow{\cong} L(b+b')$$

from two cospans

$$L(a) \xrightarrow{i} x \xleftarrow{o} L(b)$$

$$L(a') \xrightarrow{i'} x' \xleftarrow{o'} L(b')$$

via coproducts. It is shown in Theorem 3.9 that ${}_L\mathsf{Csp}(\mathsf{X})$ is a symmetric monoidal double category when A and X have finite colimits and L preserves them. Using [*M. A. Shulman*, "Constructing symmetric monoidal bicategories", Preprint, arXiv:1004.0993], it is shown in Corollary 3.10 that ${}_L\boldsymbol{Csp}(\mathsf{X})$ is a symmetric monoidal bicategory when A and X have finite colimits and L preserves them. It is shown in Corollary 3.11 that ${}_L\mathsf{Csp}(\mathsf{X})$ is a symmetric monoidal category when A and X have finite colimits and L preserves them. It is shown in Theorem 3.12 that the category ${}_L\mathsf{Csp}(\mathsf{X})$ is actually a special sort of symmetric monoidal category called a *hypergraph category* [*B. Fong* and *D. I. Spivak*, "Hypergraph categories", Preprint, arXiv:1806.08304].

§4 shows how to construct maps between structured cospan categories, bicategories or double categories.

§5 illustrates some of the advantages of structured categories over decorated ones with an example of the double category with open graphs as morphisms.

Decorated cospans have already been exploited to study electrical circuits [*J. C. Baez* and *B. Fong*, Theory Appl. Categ. 33, 1158–1222 (2018; Zbl 1402.18005)], Markov processes [*J. C. Baez* et al., J. Math. Phys. 57, No. 3, 033301, 30 p. (2016; Zbl 1336.60147)] and reaction networks [*J. C. Baez* and *B. S. Pollard*, Rev. Math. Phys. 29, No. 9, Article ID 1750028, 41 p. (2017; Zbl 1383.68053)], while structured cospans have been used to study electrical circuits [Zbl 1400.18004] and Petri nets [*J. C. Baez* and *J. Master*, "Open Petri nets", Math. Struct. Comput. Sci. 30, 314–341 (2020; doi:10.1017/S0960129520000043)]. §6 aims to show that structured cospans can take the place of decorated cospans in many of these applications.

Appendix is devoted to a brief review of double categories, referring the reader to [*M. Grandis* and *R. Paré*, Cah. Topologie Géom. Différ. Catégoriques 40, No. 3, 162–220 (1999; Zbl 0939.18007); *M. Grandis* and *R. Pare*, Cah. Topol. Géom. Différ. Catég. 58, No. 1, 3–48 (2017; Zbl 1387.18013); *L. W. Hansen* and *M. Shulman*, "Constructing symmetric monoidal bicategories functorially", Preprint, arXiv:1910.09240; *M. Shulman*, Theory Appl. Categ. 20, 650–738 (2008; Zbl 1192.18005); "Constructing symmetric monoidal bicategories", Preprint, arXiv:1004.0993] for more detailed expositions.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

18B10 Categories of spans/cospans, relations, or partial maps
18M35 Categories of networks and processes, compositionality
18N10 2-categories, bicategories, double categories

Keywords:

bicategory; cospan; double category; monoidal category; network

Full Text: Link

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