Culbertson, Jared; Gustafson, Paul; Koditschek, Daniel E.; Stiller, Peter F.
Formal composition of hybrid systems. (English) Zbl 07266040
Theory Appl. Categ. 35, 1634-1682 (2020).

This paper aims to construct a physically-grounded compositional framework for hybrid system synthesis, particularly targeted at applications in robotics. Compositionality lies at the heart of language in general [*M. Werning* et al., The Oxford handbook of compositionality. Oxford Handbooks in Linguistics (2012)], its formalization underlying much of computer science [*J. van Leeuwen* (ed.), Formal models and semantics. Handbook of theoretical computer science. Vol. B. Amsterdam etc.: Elsevier (1990; Zbl 0714.68001)]. The formalism in this paper is inspired by three distinct notions of behavioral composition, namely, *sequential composition*, *hierarchical composition* and *parallel composition*. The paper formalizes a useful subset of each of these three motivating concepts by using category theory which is particularly effective in description of composition and abstraction. What is more significant, the Curry-Howard correspondence enables one to transfer the categorical results in this paper to the setting of functional programming. Although connections to robotic behavioral programming are not explored in this paper, developing more physically grounded analogues of existing formal approaches to motion planning in use of linear-time temporal logic and functional reactive programming are involved in the authors' long-term ambitions. Similarly, the construction of exponential objects for hybrid systems categories, which is a inevitable step in constructing a full-fledged functional programming language for hybrid systems is not addressed, presumably requiring more careful consideration. This paper is the first step in this long journey, the main contribution being the investigation of a series of categories of hybrid systems that are both mathematically rigorous and also loyal to the way that robotics engineers usually approach model development. This paper was inspired by *E. Lerman*'s elegant definition of a category of hybrid systems and semiconjugacies (loosely speaking, execution-preserving maps) in [A category of hybrid systems", Preprint, arXiv:1612.01950].

Since robotics applications inevitably incur sudden transitions between dynamics and state spaces cosequent upon the making and breaking of different contacts with different portions of the environment, this paper must depart from classical theory to embrace a notion of hybrid dynamical systems. The authors are primarily interested in a formalism focused on *non-blocking* and *deterministic* [*E. Lerman* and *J. Schmidt*, "Networks of hybrid open systems", Preprint, arXiv:1908.10447] executions, which are hybrid versions of existence and uniqueness in classical dynamical systems theory.

Sequential composition formalizes notions of *pre-image back-chaining* [*T. Lozano-Perez* et al., "Automatic synthesis of fine-motion strategies for robots", Int. J. Robot. Res. 3, No. 1, 3–24 (1984)], earning wide attention in robotics as corresponding to broadly useful event-based concatenation of behavior over time. The authors formulates a notion of *directed* hybrid system, a system in which a generic execution flows from a domain subsystem to a codomain subsystem, providing a basis for modeling simple robotic behaviors as directed hybrid systems of specified initial and final interfaces available for linking behaviors to achieve more complex behaviors. It is shown that there exists a double category of hybrid systems where directed systems are the horizontal morphisms and semiconjugacies form the vertical morphisms, providing a setting for exploring both model abstraction together with sequential composition.

Hierarchical composition has a still older pedigree, based on the folklore dynamical systems *collapse of dimension* concept, deeply engrained in the literature. The authors formalize the notion of a *template-anchor pair* of hybrid systems. In dynamical systems theory, a *template* is a low degree of freedom, idealized model of a physical system, while an *anchor* corresponding to such a template is a high degree of freedom, more realistic model of the same system. A *template-anchor pair* consists of an embedding of a template model as an attracting, invariant subsystem of a corresponding anchor model. As long as trajectories in the anchor converge to the template sufficiently promptly, the dynamics of the template gives a good approximation for the dynamics of the anchor. In the continuous setting, template-anchor pairs are of a splendid and well-established theory [*J. Eldering* et al., Nonlinearity 31, No. 9, 4202–4245 (2018; Zbl 1396.37035)]. Unfortunately, mismatches between resets in templates and anchors complicate the theory in the hybrid setting. To address these complications, the authors introduce formal subdivisions of hybrid systems, namely, semiconjugacies abiding by a fiber product property for execution, allowing one to add formal resets to template systems. The authors then define a template-anchor pair as a span for which the left leg is a template, the roof is a subdivision of the template, and the right morphism embeds this subdivision into the anchor as an attracting, invariant subsystem, hierachical composition corresponding to composing spans by taking a fiber product of subdivisions over a system that is both an anchor for a simpler template and a template for a more complicated anchor.

Parallel composition, which is a simultaneous operation of distinct behaviors of the same body, has only relatively recently been achieved in an empirically reliable form suitable to highly energetic mechanisms [*M. H. Raibert*, Legged robots that balance. Cambridge: MIT Press (1986)] with a corresponding mathematical theory only burgeoning recently [*A. De* and *D. E. Koditschek*, "Parallel composition of templates for tail-energized planar hopping", in: IEEE International Conference on Robotics and Automation (ICRA), 4562–4569 (2015746); "Vertical hopper compositions for preflexive and feedback-stabilized quadrupedal bounding, pacing, pronking, and trotting", Int. J. Robot. Res. 37, No. 7, 743–778 (2018)] inspired by the challenges of circumventing destabilizing cross-talk [*D. E. Whitney*, "Why mechanical design cannot be like VLSI design", Res. Eng. Design 8, 125–138 (1996)]. As a first attempt at parallel composition, this paper concentrates on combining decoupled hybrid systems, showing that the category of hybrid systems is cartesian. A similar result is obtained in [arXiv:1908.10447], where more complex coupled systems are explored using interconnection maps. This paper is interested in connections between parallel composition and the other two forms of composition under consideration.

This review is concluded with a criticism against the authors' references. The authors refers to a paper, e.g., simply as [JBK 16] in the text, which presumably points to

Aaron M. Johnson, Samuel A. Burden, and Daniel E. Koditschek,
A hybrid systems model for simple manipulation and self-manipulation systems,
The International Journal of Robotics Research **35** (2016), no.11, 1354-1392.

in References, which is highly irritating and cumbersome to find out. Why did they hesitate to write in the References as follows.

[JBK16] Aaron M. Johnson, Samuel A. Burden, and Daniel E. Koditschek,
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Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

18B20 Categories of machines, automata
18N10 2-categories, bicategories, double categories

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hybrid dynamical system; semiconjugacy; double category

Full Text: Link

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[55] Autonomy Capabilities Team, Air Force Research Laboratory

[56] Dayton, OH 45433, USA

[57] Department of Electrical Engineering, Wright State University

[58] Dayton, OH 45435, USA

[59] School of Engineering and Applied Science, University of Pennsylvania

[60] Philadelphia, PA 19104, USA

[61] Department of Mathematics, Texas A&M University

[62] College Station, TX, 77840, USA

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[95] Joachim Kock, Universitat Aut'onoma de Barcelona:kock (at) mat.uab.cat
[96] Stephen Lack, Macquarie University:steve.lack@mq.edu.au
[97] Tom Leinster, University of Edinburgh:Tom.Leinster@ed.ac.uk
[98] Matias Menni, Conicet and Universidad Nacional de La Plata, Argentina:matias.menni@gmail.com
[99] Ieke Moerdijk, Utrecht University:i.moerdijk@uu.nl
[100] Susan Niefield, Union College:niefiels@union.edu
[101] Kate Ponto, University of Kentucky:kate.ponto (at) uky.edu
[102] Robert Rosebrugh, Mount Allison University:rrosebrugh@mta.ca
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[104] Giuseppe Rosolini, Universit'a di Genova:rosolini@disi.unige.it
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[107] Ross Street, Macquarie University:ross.street@mq.edu.au