Trotta, Davide

The existential completion. (English) Zbl 07266038

Theory Appl. Categ. 35, 1576-1607 (2020).

Many relevant logical completions have been investigated within category theory in recent years, the main instance being the exact completion [*A. Carboni*, Rapp., Sémin. Math., Louvain, Nouv. Sér. 221–236, 55–98 (1993; Zbl 0838.18003); J. Pure Appl. Algebra 103, No. 2, 117–148 (1995; Zbl 0839.18002); Zbl 0504.18005,*A. Carboni* and *R. Celia Magno*, J. Aust. Math. Soc., Ser. A 33, 295–301 (1982; Zbl 0504.18005); *R. A. Johnson* and *K. T. Wu*, Stat. Probab. Lett. 12, No. 6, 517–525 (1991; Zbl 0742.62098); Zbl 0866.34016*C. De Coster* and *M. Gaudenzi*, Rapp., Sémin. Math., Louvain, Nouv. Sér. 245–260, 315–339 (1996; Zbl 0866.34016)], which is the universal extension of a category with finite limits to an exact category. A categorical version of quotient for an equivalence relation was introduced and investigated within a doctrine of a sufficient logical structure so as to describe the notion of an equivalence relation [*M. E. Maietti* and *G. Rosolini*, Theory Appl. Categ. 27, 445–463 (2013; Zbl 1288.03048); Log. Univers. 7, No. 3, 371–402 (2013; Zbl 1288.03049); Appl. Categ. Struct. 23, No. 1, 43–52 (2015; Zbl 1386.03074)]. It was demonstrated that both the exact completion of a regular category and the exact completion of a category with binary products, a weak terminal object and weak pullbacks are to be seen as instances of a more general completion with respect to an elementary existential doctrine [*M. E. Maietti* and *G. Rosolini*, Appl. Categ. Struct. 23, No. 1, 43–52 (2015; Zbl 1386.03074)].

This paper presents the existential completion of a primary doctrine, giving an explicit description of the 2-monad

$$\mathrm{T}_e : \boldsymbol{PD} \to \boldsymbol{PD}$$

constructed from the 2-adjunction, where $\boldsymbol{PD}$ is the 2-category of primary doctrines. It is shown that every existential doctrine

$$P : \mathcal{C}^{\mathrm{op}} \to \boldsymbol{InfSL}$$

admits an action

$$a : \mathrm{T}_e\,(P) \to P$$

such that (P, a) is a T_e-algebra, and that if (R, b) is a T_e-algebra, then the doctrine is existential, giving the equivalence the 2-category of T_e-$\boldsymbol{Alg}$ and the 2-category $\boldsymbol{ED}$ whose objects are existential doctrines. It is also demonstrated that the existential completion preserves the elementary structure of a doctrine, generalizing the bi-adjunction

$$\boldsymbol{EED} \to \boldsymbol{Xct}$$

in [*M. E. Maietti* and *G. Rosolini*, Appl. Categ. Struct. 23, No. 1, 43–52 (2015; Zbl 1386.03074); *M. E. Maietti* et al., Tbil. Math. J. 10, No. 3, 141–166 (2017; Zbl 1401.03109)] to a bi-adjunction from the 2-category $\boldsymbol{EID}$ of elementary doctrines to the 2-category $\boldsymbol{Xct}$ of exact categories.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

18C10 Theories (e.g., algebraic theories), structure, and semantics
18C15 Monads (= standard construction, triple or triad), algebras for monads, homology and derived functors for monads

Keywords:

existential completion; doctrines; property-like monad; tripos

Full Text: Link

References:

[1] Blackwell, R., Kelly, G. M., and Power, J. (1989). Two-dimensional monad theory.J. Pure Appl. Algebra, 59:1-41. ·

Zbl 0675.18006

[2] Borceux, F. (1994).Handbook of Categorical Algebra 1: Basic Category Theory, volume 51 ofEncyclopedia of Mathematics and its Applications.Cambridge Univ. Press. · Zbl 0803.18001

[3] Carboni, A. (1995). Some free constructions in realizability and proof theory.J. Pure Appl. Algebra, 103:117-148. · Zbl 0839.18002

[4] Carboni, A. and Celia Magno, R. (1982). The free exact category on a left exact one. J. Aust. Math. Soc., 33:295-301. · Zbl 0504.18005

[5] Carboni, A. and Vitale, E. M. (1998). Regular and exact completions.J. Pure Appl. Algebra., 125:79-117. · Zbl 0891.18002

[6] Frey, J. (2014). A fibrational study of realizability toposes.arXiv: Category Theory.

[7] Hofstra, P. (2010). The dialectica monad and its cousins. InModels, Logics, and Higherdimensional Categories: A Tribute to the Work of Mihaly Makkai, volume 53 ofCRM Proceedings and Lecture Notes, pages 107-139. American Mathematical Society.

[8] Hughes, J. and Jacobs, B. (2003). Factorization systems and fibrations: toward a fibered birkhoff variety theorem.Electron. Notes Theor. Comp. Sci., 69:156-182. · Zbl 1270.18003

[9] Hyland, M., Johnstone, P., and Pitts, A. M. (1980). Tripos theory.Math. Proc. Camb. Phil. Soc., 88:205-232. · Zbl 0451.03027

[10] Kelly, G. M. (1992). A note on relations relative to a factorisation system. In A. Carboni, I. C. P. and Rosolini, G., editors,Category Theory '90, volume 1488, pages 249-261. Springer-Verlag. · Zbl 0736.18001

[11] Kelly, G. M. and Lack, S. (1997). On property-like structure.Theory Appl. Categ., 3(9):213-250. · Zbl 0935.18005

[12] Kelly, G. M. and Street, R. (1974). Review of the elements of 2-categories. InCategory Seminar. Lecture Notes in Mathematics, volume 420, pages 75-103. Springer.

[13] Lack, S. (2010).A 2-Categories Companion, pages 105-191. Springer New York, New York, NY. · Zbl 1223.18003

[14] Lawvere, F. W. (1969). Adjointness in foundations.Dialectica, 23:281-296. · Zbl 0341.18002

[15] Lawvere, F. W. (1970).Equality in hyperdoctrines and comprehension schema as an adjoint functor. In Heller, A., editor,New York Symposium on Application of Categorical Algebra, volume 2, page 1-14. American Mathematical Society. · Zbl 0234.18002

[16] Leinster, T. (2003).Higher Operads, Higher Categories. London Mathe-matical Society Lecture Notes Series. Cambridge Univ. Press.

[17] Maietti, M. E. (2005). Modular correspondence between dependent type theories and categories including pretopoi and topoi.Mathematical Structures in Computer Science, 15(6):1089-1149. · Zbl 1093.03042

[18] Maietti, M. E., Maneggia, P., de Paiva, V., and Ritter, E. (2005). Relating categorical semantics for intuitionistic linear logic.Applied Categorical Structures, 13:1-36. · Zbl 1067.03067

[19] Maietti, M. E., Pasquali, F., and Rosolini, G. (2017). Triposes, exact completions, and hilbert's -operator.Tbilisi Mathematica journal, 10(3):141-166. · Zbl 1401.03109

[20] Maietti, M. E. and Rosolini, G. (2013a). Elementary quotient completion.Theory App. Categ., 27(17):445-463. · Zbl 1288.03048

[21] Maietti, M. E. and Rosolini, G. (2013b).Quotient completion for the foundation of constructive mathematics.Log. Univers., 7(3):371-402. · Zbl 1288.03049

[22] Maietti, M. E. and Rosolini, G. (2013c).Unifying exact completions.Appl. Categ. Structures, 23:43-52. · Zbl 1386.03074

[23] Marmolejo, F. (1999).Distributive laws for pseudo monads.Theory Appl. Categ., 5(5):91-147. · Zbl 0919.18004

[24] Marmolejo, F. and Wood, R. J. (2008). Coherence for pseudodistributive laws revisited. Theory Appl. Categ., 20(6):74-84. · Zbl 1153.18005

[25] Pitts, A. M. (1995). Categorical logic. In Abramsky, S., Gabbay, D. M., and Maibaum, T. S. E., editors,Handbook of Logic in Computer Science, volume 6, pages 39-.129. Oxford Univ. Press.

[26] Pitts, A. M. (2002).Tripos theory in retrospect.Math. Struct. in Comp. Science, 12:265-279. · Zbl 1005.18005

[27] Tanaka, M. (2004).Pseudo-distributive laws and a unified framework for variable binding. PhD thesis, The University of Edinburgh.

[28] Tanaka, M. and Power, J. (2006a). Pseudo-distributive laws and axiomatics for variable binding.Higher-Order Symb. Comput., 19:305-337. · Zbl 1105.68077

[29] Tanaka, M. and Power, J. (2006b). A unified category-theoretic semantics for binding signatures in substructural logics.Oxford Univ. Press, 16(1). · Zbl 1105.03072

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