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Affine geometric spaces in tangent categories. (English) Zbl 07049198

Theory Appl. Categ. 34, 405-437 (2019).

An axiomatization of the tangent functor was first given in [*J. Rosický*, Diagrammes 12, JR 1-JR 11 (1984; Zbl 0561.18008)], which was elaborated on by *J. R. B. Cockett* and *G. S. H. Cruttwell* [Appl. Categ. Struct. 22, No. 2, 331–417 (2014; Zbl 1304.18031)] with additional ideas in [*J. R. B. Cockett* and *G. S. H. Cruttwell*, Cah. Topol. Géom. Différ. Catég. 56, No. 4, 301–316 (2015; Zbl 1353.18009); ibid. 59, No. 1, 10–92 (2018; Zbl 1419.18016); Theory Appl. Categ. 32, 835–888 (2017; Zbl 1374.18016); *G. S. H. Cruttwell* and *R. B. B. Lucyshyn-Wright*, J. Homotopy Relat. Struct. 13, No. 4, 867–925 (2018; Zbl 1405.18017)]. It was *B. Jubin* ["The tangent functor monad and foliations", Preprint, `arXiv:1401.0940`] that demonstrated that the tangent functor has precisely one monad structure while it has no comonad structure at all. He has shown also that there are infinite families of monads and comonads as long as one restricts the subcategory of affine manifolds and affine maps, mixed distributive laws [*B. Mesablishvili* and *R. Wisbauer*, J. K-Theory 7, No. 2, 349–388 (2011; Zbl 1239.18002)] between these structures holding. This paper aims to present an abstraction of Jubin's systems of monads and comonads.

L. Auslander and *L. Markus* [Ann. Math. (2) 62, 139–151 (1955; Zbl 0065.37603)] have shown that an affine manifold can be defined as a manifold equipped with a flat torsion-free connection on its tangent bundle, which prompted [*J. R. B. Cockett* and *G. S. H. Cruttwell*, Theory Appl. Categ. 32, 835–888 (2017; Zbl 1374.18016)]. This paper defines a *geometric space* to be an object endowed with a connection on its tangent bundle and an *affine geometric space* to be a geometric space whose associated connection is flat and torsion-free. Maps in the category of geometric spaces are those maps commuting with the given connections. It is shown that the various geometric categories considered in this paper remain tangent categories with the structure lifting from the base category. The authors give an alternative characterization of flat torsion-free connections, claiming that a flat torsion-free connection K is to be seen as a morphism in the category of geometric spaces from $T(TM)$ to TM, where $T(TM)$ and TM are endowed with the canonical geometric structures induced by K.

The authors also consider certain 2-categories of tangent categories. It is shown that there are 2-functors sending each tangent category to its tangent category of geometric spaces or affine geometric spaces. The authors demonstrate that an affine connection induces a 2-comonad on the 2-category of tangent categories. An affine tangent category is defined to be an Eilenberg-Moore coalgebra with respect to this comonad, giving an alternative characterization of these structures.

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MSC:

18D99 Categorical structures
18F15 Abstract manifolds and fiber bundles (category-theoretic aspects)
53A15 Affine differential geometry
53B05 Linear and affine connections
53C05 Connections, general theory

Keywords:

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References:

[1] L. Auslander. The structure of complete locally affine manifolds. Topology 3, pp. 131-139, (1964). AFFINE GEOMETRIC SPACES IN TANGENT CATEGORIES435 · Zbl 0136.43102

[2] L. Auslander, L. Markus. Holonomy of flat affinely connected manifolds. Annals of Mathematics 62, pp. 139–151,

(1955). · Zbl 0065.37603

[3] M. Barr, C. Wells. Toposes, Triples and Theories, Springer-Verlag, (1984). · Zbl 0567.18001

[4] K. Bauer, B. Johnson, C. Osborne, E. Riehl, and A. Tebbe, Directional derivatives and higher order chain rules for abelian functor calculus. Topology and its Applications 235, pp. 375–427, (2018). · Zbl 1435.18005

[5] F. Bergeron, G. Labelle, P. Leroux. Combinatorial Species and Tree-like Structures. Cambridge University Press, (1997). · Zbl 0888.05001

[6] R. Blackwell, G. M. Kelly, A. J. Power. Two-dimensional monad theory, Journal of Pure and Applied Algebra 59, pp. 1–41, (1989). · Zbl 0675.18006

[7] R. Blute, J.R.B. Cockett, R.A.G. Seely. Differential categories. Mathematical Structures in Computer Science 16, pp. 1049-1083, (2006). · Zbl 1115.03092

[8] R. Blute, J.R.B. Cockett, R.A.G. Seely. Cartesian differential categories. Theory and Applications of Categories 22, pp. 622–672, (2009). · Zbl 1262.18004

[9] R. Blute, T. Ehrhard, C. Tasson. A Convenient Differential Category, Cahiers de Topologie et G´eom´etrie Diff´erentielle Cat´egoriques 53, pp. 211-232, (2012). · Zbl 1281.46061

[10] M. Bunge and P. Sawyer. On connections, geodesics and sprays in synthetic differential geometry. Cahiers de Topologie et G´eom´etrie Diff´erentielle Cat´egoriques 25, pp 221–258, (1984). · Zbl 0582.18007

[11] J.R.B. Cockett, G. Cruttwell. Differential structure, tangent structure, and SDG. Applied Categorical Structures 22, pp. 331-417, (2014). · Zbl 1304.18031

[12] J.R.B. Cockett, G. Cruttwell. The Jacobi identity for tangent categories. Cahiers de Topologie et G´eom´etrie Diff´erentielle Cat´egoriques 56, pp. 301-316, (2015).

[13] J.R.B. Cockett, G. Cruttwell. Differential bundles and fibrations for tangent categories. Cahiers de Topologie et G´eom´etrie Diff´erentielle Cat´egoriques 59, pp. 10–92, 2018. · Zbl 1419.18016

[14] J.R.B. Cockett, G. Cruttwell. Connections in tangent categories. Theory and Applications of Categories 32, pp. 835-888 (2017). · Zbl 1374.18016

[15] G. Cruttwell. Forms and exterior differentiation in Cartesian differential categories. Theory and Applications of Categories 28, pp. 981-1001, (2013). · Zbl 1288.18008

[16] G. S. H. Cruttwell, R. B. B. Lucyshyn-Wright. A simplicial foundation for differential and sector forms in tangent categories. Journal of Homotopy and Related Structures 13, pp. 867–925, (2018). 436R. F. BLUTE, G. S. H. CRUTTWELL, AND R. B. B. LUCYSHYN-WRIGHT · Zbl 1405.18017

[17] C. Dodson, T. Poston. Tensor Geometry: The Geometric Viewpoint and its Uses. Springer-Verlag, (2009). · Zbl 0369.53012

[18] T. Ehrhard, L. Regnier The differential -calculus. Theoretical Computer Science 309 pp. 1–41, (2003). · Zbl 1070.68020

[19] T. Ehrhard, L. Regnier. Differential interaction nets. Workshop on Logic, Language, Information and Computation (WoLLIC), invited paper. Electronic Notes in Theoretical Computer Science 123, March 2005, Elsevier.

[20] A.Fr¨olicher, A. Kriegl. Linear Spaces and Differentiation Theory, Wiley, (1988).

[21] R. Garner, An embedding theorem for tangent categories. Advances in Mathematics 323, pp. 668–687, (2018). · Zbl 1430.18007

[22] J.-Y. Girard, Linear logic. Theoretical Computer Science 50, pp. 1–102, (1987). · Zbl 0625.03037

[23] W. Goldman. Two examples of affine manifolds. Pacific Journal of Mathematics 94, pp. 327–330, (1981). · Zbl 0461.53041

[24] W. Goldman. Complete affine manifolds: A survey. Preprint found at http://www.math.umd.edu/ wmg, (2013).

[25] N. Hicks. Connexion preserving, conformal, and parallel maps. Michigan Mathematical Journal 10, pp. 295–302, (1963). · Zbl 0178.55801

[26] B. Jubin. The Tangent Functor Monad and Foliations. Ph.D. Thesis, University of California, Berkeley (2012). Available at arXiv:1401.0940.

[27] A. Kock. Synthetic Differential Geometry, Second Edition, Cambridge University Press, (2006). · Zbl 1091.51002

[28] I. Kolar, P. Michor, and J. Slovak. Natural Operations in Differential Geometry. Springer-Verlag, (1993). · Zbl 0782.53013

[29] A. Kriegl, P. Michor. The Convenient Setting of Global Analysis. American Mathematical Society, (1997). · Zbl 0889.58001

[30] S. Lang. Fundamentals of differential geometry. Springer-Verlag, (1999). · Zbl 0932.53001

[31] P. Leung. Classifying tangent structures using Weil algebras. Theory and Applications of Categories 32, pp. 286–337, (2017). · Zbl 1357.18003

[32] R. B. B. Lucyshyn-Wright. On the geometric notion of connection and its expression in tangent categories. Theory and Applications of Categories 33, pp. 832–866, (2018). · Zbl 1408.18030

[33] B. Mesablishvili, R. Wisbauer. Bimonads and Hopf monads on categories. Journal of K-Theory 7, pp. 349–388, (2011). AFFINE GEOMETRIC SPACES IN TANGENT CATEGORIES437 · Zbl 1239.18002

[34] K. Nomizu, T. Sasaki. Affine Differential Geometry. Cambridge University Press, (1994). · Zbl 0834.53002

[35] L. Patterson. Connexions and prolongations. Canadian Journal of Mathematics 27, pp. 766-791, (1975). · Zbl 0307.53018

[36] J. Rosick´y. Abstract tangent functors. Diagrammes 12, Exp. No. 3, (1984). · Zbl 0561.18008

[37] J. Szilasi, R. Lovas, and D. Kert´esz. Connections, sprays, and Finsler structures. World Scientific, (2014). · Zbl 1348.53002

