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Jean-Louis Koszul and the elementary structures of information geometry. (English)

Zbl 07078881

Nielsen, Frank (ed.), Geometric structures of information. Proceedings of the conference on geometric science of information, GSI 2017. Cham: Springer (ISBN 978-3-030-02519-9/hbk; 978-3-030-02520-5/ebook). Signals and Communication Technology, 333-392 (2019).

French engineer François Massieu (1832–1896) presented an idea to derive some mechanical and thermal properties of physical systems from characteristic functions in 1869 [JFM 02.0826.01], which was developed by American physicist Willard Gibbs (1839–1903) and French theoretical physicist Pierre Maurice Marie Duhem (1861–1916) [JFM 23.1172.01; JFM 24.1096.03; Journal de Mathématiques pures et appliquées 10, 207–286 (1894); Revue des deux Mondes 130, 851–868 (1895)] in thermodynamics and which was introduced into probability by French mathematician Jules-Henri Poincaré (1854–1912) [JFM 27.0190.11]. Massieu's 1869 paper has been the source of the mathematical conception of the energy of a system being equal to summations of the products of pairs of conjugate variables.

On the one hand, the Koszul-Vinberg characteristic function (KVCF) on convex cones is the cornerstone of information geometry, Koszul entropy and the Fisher information metric being defined as the Legendre transform of minus logarithm of KVCF and the Hessian of these dual functions invariant by their automorphisms, respectively. On the other hand, Souriau extended the characteristic function in statistical physics, seeking after other kinds of invariances through coadjoint action of a group on its momentum spaces and defining physical observables such as energy, heat and momentum as genuinely geometric objects. In covariant Souriau model, Gibbs equilibriums are indexed by a geometric parameter, the geometric temperature with values in the Lie algebra of the dynamical Galileo/Poincaré group being interpreted as a space-time vector, which gives a null Lie derivative to the metric tensor.

It was Fréchet's paper [Zbl 0060.30702] that first introduced the Clairaut-Legendre equation (fundamental equation in information geometry) and Fisher metric as the Hessian of a convex function. Fréchet's seminal work was followed by Koszul's two papers in the 1950s [Zbl 0066.16104; Zbl 0097.37102], which introduced forms as a generalization of Fisher metric for sharp convex cones. It was in 1969 that Souriau completed this extension within the framework of the Lie group thermodynamics with a cohomological definition of Fisher metric, which was developed by Koszul at the beginning of the 1980's in his lecture [Zbl 1433.53002]. Strange enough, many researchers on information geometry did their work without being conscious of Koszul's significant work at all, presumably because of their unfamiliarity with representation theory introduced by KIllirov [Zbl 0342.22001], in particular with affine representations of Lie groups and Lie algebras. One of these bad examples is Shun-ichi Amari [Zbl 1350.94001; Zbl 0559.62001]. Barbaresco is keenly aware of the significance of Koszul's work in information geometry so that he stresses its importance here and there [Zbl 1338.94028; Zbl 1406.94011; "Geometric theory of heat from Souriau Lie groups thermodynamics and Koszul Hessian geometry: applications in information geometry for exponential families", Entropy 18, 386 (2016)].

Koszul was gone in January 2018, and this paper pays tribute to a part of Professor Jean-Louis Koszul's work in the field of information geometry, which has many applications in the domain of applied mathematics and in the engineering applications of artificial intelligence where the most efficient and robust algorithms are based upon the natural gradient of information geometry deduced from Fisher metric, as was demonstrated in [Zbl 1428.82047; arXiv:1712.08449]. The paper consists of seven sections together with an appendix.

Through the study of the geometry of bounded homogeneous domains initiated by Elie Cartan [Zbl 0011.12302; JFM 56.0371.02], Koszul found out that the elementary structures are associated with Hessian manifolds on sharp convex cones [Zbl 0066.16104; Zbl 0097.37102; Zbl 0173.50001; Zbl 0144.34002; Zbl 0191.20501; Zbl 0195.04605; Zbl 0167.50103; Zbl 0213.36002], with which §3 is concerned. Koszul was once a Ph. D. student of Henri Cartan, but he was influenced more by Elie Cartan.

Koszul Hessian geometry structures are the key tool to define elementary structures of information geometry. §4 addresses links between Koszul-Vinberg characteristic function, Koszul forms and information

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geometry.

§5 is concerned with Koszul's study of homogeneous bounded domains and affine representations of Lie groups and Lie algebras. Koszul worked in collaboration with his student Jacques Vey [Zbl 0155.30602; Zbl 0206.51302]. Koszul developed his theory of homogeneous domains, particularly studying the homogeneous symmetric bounded domains of Siegel [Zbl 0138.31403; Zbl 0012.19703]. Koszul investigated symmetric homogeneous spaces, scrutinizing the relation of invariant flat affine connections, the affine representations of Lie algebras and invariant Hessian metrics characterized by affine representations of Lie algebras.

§6 is concerned mainly with [Zbl 1433.53002], and §7 is conclusion. Appendix is engaged in Clairaut-Legendre equation of Maurice Fréchet associated to distinguished functions as fundamental equation of information geometry.

For the entire collection see [Zbl 1407.62027].

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

94A17 Measures of information, entropy
22E30 Analysis on real and complex Lie groups
17B67 Kac-Moody (super)algebras; extended affine Lie algebras; toroidal Lie algebras
53C15 General geometric structures on manifolds (almost complex, almost product structures, etc.)
60D99 Geometric probability and stochastic geometry

Keywords:

Koszul-Vinberg characteristic function; Koszul forms; affine representation of Lie algebra and Lie group; homogeneous bounded domains

Full Text: DOI

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