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Model category of diffeological spaces. (English summary)
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The principal objective in this paper is to endow the category $\mathcal{D}$ of diffeological spaces with a compactly generated model structure, in which weak equivalences are just smooth maps inducing isomorphisms on smooth homotopy groups [J. P. May and K. Ponto, *More concise algebraic topology*, Chicago Lectures in Math., Univ. Chicago Press, Chicago, IL, 2012 (Definition 15.2.1); MR2884233]. Since every object is fibrant with respect to the author's model structure (Theorem 1.3), cofibrant diffeological spaces play a significant role in smooth homotopy theory, just as CW-complexes do in topological homotopy theory [H. Kihara, "Quillen equivalences between the model categories of smooth spaces, simplicial sets, and arc-gengerated spaces", preprint, (§1) arXiv:1702.04070]. Since the standard p-simplex Δ^p endowed with the sub-diffeology of $\mathbb{R}^{p+1}$ does not contain the kth horn Λ^p_k as a deformation retract, some technical tricks are indispensable for constructing a model structure on $\mathcal{D}$ after the case of topological spaces [P. S. Hirschhorn, *Model categories and their localizations*, Math. Surveys Monogr., 99, Amer. Math. Soc., Providence, RI, 2003 (Definition 7.10.6 and Example 11.1.8); MR1944041]. The paper [J. D. Christensen and E. Wu, New York J. Math. **20** (2014), 1269–1303; MR3312059] defined fibrations, cofibrations and weak equivalences, making use of affine p-spaces in place of standard p-simplices [J. D. Christensen and E. Wu, op. cit. (Definition 4.8)], to conjecture that $\mathcal{D}$ is a model category with these constructions. T. Haraguchi and K. Shimakawa in Theorem 5.1 of ["A model structure on the category of diffeological spaces", preprint, arXiv:1311.5668] put to use the notion of a tame map (Definition 3.8 of that paper) to define fibrations, and claimed that $\mathcal{D}$ is a model category which is not cofibrantly generated, only to find out that there exists a serious gap in the proof. The author establishes the Quillen equivalence between the model categories of diffeological spaces, simplicial sets and arc-generated spaces by making use of adjoint pairs provided in this paper.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.