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Formal exponential map for graded manifolds. (English summary)
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The principal objective of this paper is to introduce, for every $\mathbb{Z}$-graded manifold, a formal exponential map in a purely algebraic way and investigate its properties with applications. Although the geodesic exponential map $\exp\colon T_M \to M \times N$ associated to an affine connection ∇ on a smooth manifold M fails to transpose straightforwardly to the graded manifold context, its fiber-wise infinite-order jet evaluated along the zero section of T_M admits a genuinely algebraic description carrying over to the $\mathbb{Z}$-graded context. It is established (Theorem 4.3) that the formal exponential map $\mathrm{pbw}\colon \Gamma(S(T_M)) \to \mathcal{U}(T_M)$ is an isomorphism of filtered coalgebras over $\mathcal{C}^\infty(M)$. As applications, the authors give a much more transparent proof of the Emmrich-Weinstein theorem [C. Emmrich and A. D. Weinstein, in *Lie theory and geometry*, 217–239, Progr. Math., 123, Birkhäuser Boston, Boston, MA, 1994; MR1327535] for graded manifolds and a proof based on homological perturbation of an analog of V. A. Dolgushev's result in [Adv. Math. **191** (2005), no. 1, 147–177; MR2102846] using B. V. Fedosov's iterative method [J. Differential Geom. **40** (1994), no. 2, 213–238; MR1293654] in the context of $\mathbb{Z}$-graded manifolds.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.