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The unity and identity of decidable objects and double-negation sheaves. (English summary)

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C. McLarty proposed in [J. Symbolic Logic **52** (1987), no. 1, 202–204; MR0877866; J. Philos. Logic **17** (1988), no. 1, 75–90; MR0925615] to consider 2-valued toposes $\mathcal{E}$ with global support and such that every object X in $\mathcal{E}$ has a unique subobject $\beta_X\colon CX \to X$ with CX decidable and every global element of X factors through β_X. The principal objective in this paper is to show that the conclusion of McLarty's result holds for any stably precohesive topos $p\colon \mathcal{E} \to \mathcal{S}$ in the sense of [F. W. Lawvere and M. Menni, Theory Appl. Categ. **30** (2015), Paper No. 26, 909–932; MR3365705] with Boolean codomain. Specifically, given a topos $\mathcal{E}$, consider the following four statements:

(1) The topos $\mathcal{S}$ is Boolean and $p\colon \mathcal{E} \to \mathcal{S}$ is a stably precohesive geometric morphism.
(2) The subcategories $\mathrm{Dec}(\mathcal{E}) \to \mathcal{E}$ and $\mathcal{E}_{\neg\neg} \to \mathcal{E}$ are the left and right inclusions of a UIAO (unity and identity of adjointly opposites) [F. W. Lawvere, Appl. Categ. Structures **4** (1996), no. 2-3, 167–174; MR1406096].
(3) The inclusion has a right adjoint that reflects the initial object.
(4) The inclusion $\mathcal{E}_{\neg\neg} \to \mathcal{E}$ of $\neg\neg$ sheaves is the right inclusion of a UIAO.

By Proposition 4.4 of [F. W. Lawvere and M. Menni, op. cit.], the first item implies the fourth, while the second trivially implies the fourth. This paper proves that the second and third items are equivalent, and that they are both implied by the first.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.