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Locally anisotropic toposes. (English summary)
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Isotropy theory for toposes originated in the theory of inverse semigroups and étale groupoids [J. R. Funk, P. J. W. Hofstra and B. Steinberg, Theory Appl. Categ. **26** (2012), No. 24, 660–709; MR3065939]. In the paper under review the authors introduce isotropy torsors, showing that they are classified by étale splittings of the isotropy quotient (Proposition 4.11 and Theorem 4.12). The authors establish a structure theorem for locally anisotropic toposes, claiming that a topos is locally anisotropic if and only if it can be covered

$$\mathcal{E}/O \twoheadrightarrow \mathcal{E}$$

by an isotropically trivial object O with

$$\mathcal{E}/O = \mathcal{B}(\mathcal{F};G)$$

for some group G internal to an anisotropic topos $\mathcal{F}$, where $\mathcal{B}(\mathcal{F};G)$ is the topos of right G-objects in $\mathcal{F}$ (Theorem 4.15), in the process of which the authors develop a significant amount of general theory (by way of example, Lemma 4.3 [the fundamental lemma of isotropy theory] and Theorem 3.11 concerning the isotropy group Z_G of $\mathcal{B}(\mathcal{E};G)$ and its universal action). §4.4 explains such results in terms of isotropy algebras, which are essentially no other than discrete fibrations on the isotropy topos groupoid of $\mathcal{E}$, showing how free isotropy algebras are intimately related to the structure theorem. §5 gives a detailed explanation of how the theory is interpreted for inverse semigroups, yielding the so-called Billhardt theory [M. V. Lawson, *Inverse semigroups*, World Sci. Publ., River Edge, NJ, 1998; MR1694900] as an exemplification of the topos theory results.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.