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**Cosheafification.** (English summary)

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The principal objective of this paper is to prepare a foundation for future homology and homotopy theories of cosheaves. The author shows that if one allows precosheaves to take values in a larger category  $\mathbf{Pro}(\mathbf{K})$  [cf. M. Kashiwara and P. Schapira, *Categories and sheaves*, Grundlehren Math. Wiss., 332, Springer, Berlin, 2006 (Definition 6.1.1); [MR2182076](#); M. Artin and B. C. Mazur, *Etale homotopy*, Lecture Notes in Mathematics, No. 100, Springer, Berlin, 1969 (Appendix); [MR0245577](#); S. Mardešić and J. Segal, *Shape theory*, North-Holland Mathematical Library, 26, North-Holland, Amsterdam, 1982 (Remark I.1.4); [MR0676973](#)] in place of an arbitrary cocomplete category  $\mathbf{K}$ , the usual two-step process of the well-known plus construction  $(\ )_+$  yields the desired cosheafification. The author [*Topology Appl.* **159** (2012), no. 5, 1339–1356; [MR2879363](#)] has already developed the same approach to the case that  $\mathbf{K} = \mathbf{Set}$  and  $\mathbf{K} = \mathbf{Ab}$ . The author gives necessary and sufficient conditions for smoothness of a precosheaf with values in the original category  $\mathbf{K}$ , while precosheaves with values in  $\mathbf{Pro}(\mathbf{K})$  are always smooth. Constant cosheaves are constructed, which has much to do with shape theory. The author suggests that he will develop homology of cosheaves. *Hirokazu Nishimura*

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*Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.*