MR3648989 18B25 18B40 20L05 20M18

Kudryavtseva, Ganna (SV-LJUBCE); **Škraba, Primož [Skraba, Primoz]** (SV-IJS)

The principal bundles over an inverse semigroup. (English summary)

Semigroup Forum **94** (2017), *no. 3,* 674–695.

In [Theory Appl. Categ. **24** (2010), No. 6, 117–147; MR2720180], J. R. Funk and P. J. W. Hofstra proposed a definition of an S-torsor with an inverse semigroup S in an arbitrary Grothendieck topos $\mathcal{E}$, claiming an equivalence of categories between S-torsors in $\mathcal{E}$ and filtered functors $L(S) \to \mathcal{E}$ with Loganathan's category $L(S)$, even though [op. cit.] suffers from some deficiencies (e.g., the exact definition of action by partial bijections in an arbitrary Grothendieck topos is missing). One of the principal objectives in this paper is to make the constructions in [op. cit.] as detailed, simple and explicit as possible, and particularly tailored to researchers in semigroup theory. The authors introduce a class of connected non-strict S-sets, establishing that they are in a categorical equivalence with torsion-free functors on $L(S)$ (Theorem 10). They then show that connected non-strict S-sets form a proper coreflexive subcategory of the category of all non-strict S-sets, which corrects Proposition 3.6 of [op. cit.] (Example 7 and Proposition 12). The connection of transitive and universal S-sets with appropriate classes of functors on $L(S)$ is also discussed, leading to a new perspective on the classical result of [B. M. Schein, Izv. Vysš. Učebn. Zaved. Matematika **1962**, no. 3 (28), 164–176; MR0139674] on transitive and effective representations of an inverse semigroup. The authors define S-torsors in the topos of sheaves $\mathbf{Sh}(X)$ over a topological space X, establishing that they are categorically equivalent to filtered functors $L(S) \to \mathbf{Sh}(X)$ (Theorem 17). In the last section (§5, entitled "Towards actions of inverse semigroups in an arbitrary topos"), the authors start from a functor on $L(S)$, construct an object of action as a certain colimit and then lift S to the topos $\mathcal{H}$-class-wise, which is substantially different from the methods in [J. R. Funk and P. J. W. Hofstra, op. cit.]. *Hirokazu Nishimura*

References

1. Awodey, S.: Category Theory. Oxford University Press, Oxford (2006) MR2229319
2. Funk, J.: Semigroups and toposes. Semigroup Forum **75**(3), 481–520 (2007) MR2353278
3. Funk, J., Hofstra, P.: Topos theoretic aspects of semigroups actions. Theory Appl. Categ. **24**(7), 117–147 (2010) MR2720180
4. Funk, J., Lawson, M.V., Steinberg, B.: Characterizations of Morita equivalent inverse semigroups. J. Pure Appl. Algebra **215**, 2262–2279 (2011) MR2786616
5. Funk, J., Steinberg, B.: The universal covering of an inverse semigroup. Appl. Categ. Struct. **18**(2), 135–163 (2010) MR2601960
6. Howie, J.M.: Fundamentals of Semigroup Theory. The Clarendon Press, Oxford University Press, New York (1995) MR1455373
7. Kudryavtseva, G., Lawson, M.V.: The classifying space of an inverse semigroup. Period. Math. Hung. **70**(1), 122–129 (2015) MR3315287
8. Lawson, M.V.: Inverse Semigroups: The Theory of Partial Symmetries. World Scientific, River Edge (1998) MR1694900
9. Lawson, M.V., Margolis, S.W., Steinberg, B.: The étale groupoid of an inverse semigroup as a groupoid of filters. J. Aust. Math. Soc. **94**, 234–256 (2014) MR3109745

10. Leech, J.: $\mathcal{H}$-coextensions of monoids. Mem. Am. Math. Soc.**1**(2–15) (1975), 1–66 MR0376919
11. Loganathan, M.: Cohomology of inverse semigroups. J. Algebra **70**, 375–393 (1981) MR0623815
12. Mac Lane, S.: Categories for the Working Mathematician. Graduate Texts in Mathematics, vol. 5. Springer, New York (1998) MR1712872
13. Mac Lane, S., Moerdijk, I.: Sheaves in Geometry and Logic. A First Introduction to Topos Theory. Springer, New York (1994) MR1300636
14. Moerdijk, I.: Classifying Spaces and Classifying Topoi. Lecture Notes in Mathematics, vol. 1616. Springer, Berlin (1995) MR1440857
15. Schein, B.M.: Representations of generalized groups. Izv. Vyssh. Uchebn. Zaved Mat. **28**(3), 164–176 (1962) **(in Russian)** MR0139674
16. Steinberg, B.: Strong Morita equivalence of inverse semigroups. Houst. J. Math. **37**, 895–927 (2011) MR2844456

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.