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Complex supermanifolds of odd dimension beyond 5. (English summary)

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It is well known that complex supermanifolds appear as deformations of split complex supermanifolds $(M, \mathcal{O}_{\Lambda E})$ with $E \to M$ being a complex vector bundle and $\mathcal{O}_{\Lambda E}$ standing for the sheaf of holomorphic sections of ΛE. It is also well known that these deformations of a split complex supermanifold are to be parametrized by $H^0(M, \mathrm{Aut}(E))$-orbits in a non-abelian first Čech cohomology $H^1(M, G_E)$ [P. S. Green, Proc. Amer. Math. Soc. **85** (1982), no. 4, 587–590; MR0660609]. Its cocycles appear as exponentials of nilpotent derivations u in $C^1(M, \mathrm{Der}^2(\Lambda E))$ with $\mathrm{Der}^2(\Lambda E)$ standing for the even derivations of $\mathcal{O}_{\Lambda E}$ that increase the degree by at least two [M. J. Rothstein, Proc. Amer. Math. Soc. **95** (1985), no. 2, 255–260; MR0801334]. This paper focuses on the following two questions in connection with the computation of a suitable u.

(1) Is it possible to express the non-abelian cocycle condition on up to non-abelian coboundaries as conditions in the abelian cohomology given by $H^1(M, \mathrm{Der}^2(\Lambda E))$? This paper answers the question for split complex supermanifolds with no global even vector fields that increase the degree by two or more.

(2) The $\mathbb{Z}$-grading of $\mathcal{O}_{\Lambda E}$ induces a $\mathbb{Z}$-grading on $\mathrm{Der}^2(\Lambda E)$ so that u is the finite sum $u_2 + u_4 + u_6 + \cdots$. What are the necessary and sufficient conditions for a sum $u_2 + \cdots + u_{2q-2}$ with $2 \leq 2q \leq \mathrm{rank}(E)$ to be extendable to a $u \in C^1(M, \mathrm{Der}^2(\Lambda E))$ that defines a supermanifold structure? Necessary conditions for a recursive construction of these cochains of derivations are analyzed up to terms of degree six in this paper.

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References

1. Paul Green, *On holomorphic graded manifolds*, Proc. Amer. Math. Soc. **85** (1982), no. 4, 587–590, DOI 10.2307/2044071. MR660609 MR0660609
2. M. Kalus, *Complex supermanifolds of low odd dimension and the example of the complex projective line*, arXiv:1405.5065
3. A. L. Onishchik, *On the classification of complex analytic supermanifolds,* Towards 100 years after Sophus Lie (Kazan, 1998), Lobachevskii J. Math. **4** (1999), 47–70 (electronic). MR1743145 MR1743145
4. Mitchell J. Rothstein, *Deformations of complex supermanifolds*, Proc. Amer. Math. Soc. **95** (1985), no. 2, 255–260, DOI 10.2307/2044523. MR801334 MR0801334

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.