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**Differential geometry via infinitesimal displacements. (English summary)**

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In synthetic differential geometry [see A. B. Kock, *Synthetic differential geometry*, second edition, London Math. Soc. Lecture Note Ser., 333, Cambridge Univ. Press, Cambridge, 2006; MR2244115] tangent vectors are regarded as infinitesimal displacements and vector fields are looked upon as infinitesimal transformations. This paper presents a similar approach from the standpoint of nonstandard analysis. A former application of nonstandard analysis to differential geometry is [R. M. M. de Almeida, V. M. C. Neves and K. D. Stroyan, Differ. Geom. Dyn. Syst. **16** (2014), 1–13; MR3226602]. A serious comparison between synthetic differential geometry and nonstandard differential geometry might presumably be intriguing.

Summary: "We present a new formulation of some basic differential geometric notions on a smooth manifold $M$, in the setting of nonstandard analysis. In place of classical vector fields, for which one needs to construct the tangent bundle of $M$, we define a *prevector field*, which is an internal map from ${}^*M$ to itself, implementing the intuitive notion of vectors as infinitesimal displacements. We introduce regularity conditions for prevector fields, defined by finite differences, thus purely combinatorial conditions involving no analysis. These conditions replace the more elaborate analytic regularity conditions appearing in previous similar approaches, e.g. by Stroyan and Luxemburg or Lutz and Goze. We define the flow of a prevector field by hyperfinite iteration of the given prevector field, in the spirit of Euler's method. We define the Lie bracket of two prevector fields by appropriate iteration of their commutator. We study the properties of flows and Lie brackets, particularly in relation with our proposed regularity conditions. We present several simple applications to the classical setting, such as bounds related to the flow of vector fields, analysis of small oscillations of a pendulum, and an instance of Frobenius' Theorem regarding the complete integrability of independent vector fields."

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Current version of review. Go to earlier version. *Hirokazu Nishimura*