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**A survey on Frölicher spaces. (English summary)**

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Smooth manifolds occupy a central position in finite-dimensional differential geometry. It is well known that such an easy infinite-dimensional generalization of a smooth manifold as Banach manifolds, the Hilbert manifold or Fréchet manifolds does not suffice for truly infinite-dimensional differential geometry. In particular, we require that the category of would-be objects of infinite-dimensional differential geometry and appropriately defined smooth maps between them should be Cartesian-closed. A readable account on the comparison among various approaches to this end from a standpoint of category theory can be seen in [A. E. Stacey, *Theory Appl. Categ.* **25** (2011), No. 4, 64–117; [MR2805746](#)]. There are two outstanding approaches to this end. One is called *diffeology*, and is based on Chen-Souriau lines and now enjoys a voluminous textbook written by a former student of Souriau [P. Iglesias-Zemmour, *Diffeology*, Math. Surveys Monogr., 185, Amer. Math. Soc., Providence, RI, 2013; [MR3025051](#)]. The other is founded on the ideas of Frölicher [A. Frölicher, in *Category theory (Gummersbach, 1981)*, 69–81, Lecture Notes in Math., 962, Springer, Berlin, 1982; [MR0682945](#); K. T. Chen, *Bull. Amer. Math. Soc.* **83** (1977), no. 5, 831–879; [MR0454968](#)], and the objects under study by this approach are called *Frölicher spaces*, after him. This is a readable and comprehensive survey on the study of Frölicher spaces. *Hirokazu Nishimura*

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*Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.*