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MR3221297 53D99 18D05 18F15 58A99
Joyce, Dominic [Joyce, Dominic D.] (4-OX)

★**An introduction to d-manifolds and derived differential geometry. (English summary)**

Moduli spaces, 230–281, *London Math. Soc. Lecture Note Ser.*, 411, *Cambridge Univ. Press*, *Cambridge*, 2014.

The original motivation for *derived algebraic geometry* was the fact that certain moduli spaces $\mathcal{M}$ appearing in enumerative invariant problems might be realized as a 1-categorical truncation of a certain *derived* moduli space $\boldsymbol{M}$ inhabiting a certain higher category. Early work on derived algebraic geometry was concerned with *dg-schemes* [I. Ciocan-Fontanine and M. M. Kapranov, Ann. Sci. École Norm. Sup. (4) **34** (2001), no. 3, 403–440; MR1839580], which have been replaced by derived stacks [B. Toën, in *Algebraic geometry—Seattle 2005. Part 1*, 435–487, Proc. Sympos. Pure Math., 80, Part 1, Amer. Math. Soc., Providence, RI, 2009; MR2483943; B. Toën and G. Vezzosi, Mem. Amer. Math. Soc. **193** (2008), no. 902, x+224 pp.; MR2394633] and structured spaces [J. Lurie, *Higher topos theory*, Ann. of Math. Stud., 170, Princeton Univ. Press, Princeton, NJ, 2009; MR2522659; "Derived algebraic geometry V: Structured spaces", preprint, arXiv:0905.0459]. The author is now writing a book on *derived differential geometry* [*D-manifolds and d-orbifolds: a theory of derived differential geometry*, people.maths.ox.ac.uk/~joyce/dmanifolds.html], and this paper is intended as a survey of it.

The author is active in symplectic differential geometry, many important areas of which—including Gromov-Witten invariants, Lagrangian Floer cohomology, symplectic field theory, contact homology and Fukaya categories—are concerned with moduli spaces $\overline{\mathcal{M}}_{g,m}(J,\beta)$ of J-holomorphic curves in some symplectic manifolds (M,ω) possibly with boundary in a Lagrangian L in order to get invariants with considerably interesting properties. In order to count these moduli spaces, one is forced to impose an appropriate geometric structure, which might be called a derived moduli space, on $\overline{\mathcal{M}}_{g,m}(J,\beta)$. However, derived differential geometry is in a mess, there being no agreement on what geometric structure to exploit.

Inspired by [J. Lurie, op. cit.; D. I. Spivak, Duke Math. J. **153** (2010), no. 1, 55–128; MR2641940] but daunted by their formidable formality, the author considers a 2-category truncation of Spivak's *derived manifolds*, called d-manifolds, as well as d-manifolds with boundary or corners and their orbifolds versions, though this introduction is concerned almost exclusively with d-manifolds for the sake of brevity. After a readable introduction (§1), a survey of $\mathcal{C}^\infty$-algebraic geometry is given in §2. In §3 the author discusses what are derived $\mathcal{C}^\infty$-schemes, which are to be called d-spaces. The main topic in §4 is d-manifolds and their differential geometry. The concluding three subsections of the section are concerned with d-manifolds with boundaries or corners and their orbifold versions (§4.9), d-manifold bordism and virtual classes for d-manifolds and d-orbifolds (§4.10), and the relationship between d-manifolds and d-orbifolds and other classes of geometric spaces in the literature (§4.11). An appendix on basics of 2-categories is given.

{For the collection containing this paper see MR3236896}

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