Moeller, Joe; **Vasilakopoulou, Christina**

Monoidal Grothendieck construction. (English) Zbl 07229468

Theory Appl. Categ. 35, 1159-1207 (2020).

The Grothendieck construction gives the equivalence between contravariant pseudofunctors into Cat and fibrations, for which see, e.g., Chapter 1 of [*B. Jacobs*, Categorical logic and type theory. Amsterdam: Elsevier (1999; Zbl 0911.03001)]. This paper aims to extend this correspondence to the monoidal setting.

The paper consists of five sections as well as an appendix. §2 reviews the basic theory of fibrations and indexed categories, as well as that of monoidal 2-categories and pseudomonoids.

§3 gives explicit description of the 2-categories of pseudomonoids in the cartesian monoidal 2-categories of fibrations and indexed categories, Fib and ICat, exhibiting their equivalence induced by the *monoidal* Grothendieck construction. The authors also consider the fixed-base case, namely pseudomonoids in $\mathsf{Fib}(\mathcal{X})$ and $\mathsf{ICat}(\mathcal{X})$ as well as their corresponding equivalence. These two cases are distinct, which is summarized in

$$\begin{tikzcd}
& \mathsf{Fib} \simeq \mathsf{ICat} \arrow[dl, "\mathsf{PsMon}(-)"'] \arrow[dr, "\mathrm{fix}\,\mathcal{X}"] & \\
\mathsf{MonFib} \simeq \mathsf{MonICat} \arrow[d, "\mathrm{fix}\,\mathcal{X}"'] & & \mathsf{Fib}(\mathcal{X}) \simeq 2\mathsf{Cat}_{\mathrm{ps}}(\mathcal{X}^{\mathrm{op}}, \mathsf{Cat}) \arrow[d, "\mathsf{PsMon}(-)"] \\
\mathsf{MonFib}(\mathcal{X}) \simeq \mathsf{Mon}2\mathsf{Cat}_{\mathrm{ps}}(\mathcal{X}^{\mathrm{op}}, \mathsf{Cat}) & & \mathsf{PsMon}(\mathsf{Fib}(\mathcal{X})) \simeq 2\mathsf{Cat}_{\mathrm{ps}}(\mathcal{X}^{\mathrm{op}}, \mathsf{MonCat})
\end{tikzcd}$$

§4 shows that the two feet in the above diagram turn out to coincide under certain hypotheses, which reveals some interesting subtleties on the potential monoidal structures on fibrations and pseudofunctors. *M. Shulman* [Theory Appl. Categ. 20, 650–738 (2008; Zbl 1192.18005)] constructed an equivalence between monoidal fibrations over a cartesian monoidal base and ordinary pseudofunctors into MonCat, which motivated an investigation regarding a *fiberwise* monoidal structure of a fibration as opposed to a *global* one.

§5 is devoted to exploring certain settings where the equivalence monoidal fibrations and monoidal indexed categories naturally arises. The appendix details the description of the (braided/symmetric) monoidal structures on the total category of the Grothendieck construction, assuming the appropriate data is present. A hand-on correspondence underlying the proof as regards the transfer of the monoidal structure from a functor to its target and vice versa is also provided.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

18D30 Fibered categories
18M05 Monoidal categories, symmetric monoidal categories

Keywords:

fibrations; indexed categories; Grothendieck construction; monoidal 2-categories; monoidal pseudofunctors

Full Text: Link

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