LETTER

Synthesis of a Complex Prototype Ladder Filter Excluding Inductors with Finite Transmission Zeros Suitable for Fully Differential Gm-C Realization

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SUMMARY In this letter, an active complex filter with finite transmission zeros is proposed. In order to obtain a complex prototype ladder filter including no inductors, a new circuit transformation is proposed. This circuit is classified into the $\mathrm{R^iCR}$ filter. It is shown that it includes no negative capacitors when it is obtained through a frequency transformation. The validity of the proposed method is confirmed through computer simulation.
key words: *complex, active filter, OTA, equivalent circuit*

1. Introduction

Recently, many complex coefficient filters (complex filters) have been proposed in the field of analog signal processing [1]–[6]. Because complex filters have asymmetrical frequency characteristics with respect to DC, they are used for communication systems with orthogonal signals. Among active complex filters, complex filters realized by using OTA's and capacitors are attractive from the viewpoints of their tunability [6].

In many cases, the complex prototype ladder filters include inductors [2]. The inductors should be realized by using active elements and capacitors in the integrated circuits. Therefore, the number of the required active elements tends to become large when the complex prototype ladder filter includes inductors.

In order to reduce the active elements, a complex prototype ladder filter with finite transmission zeros has been proposed [3]. This prototype filter includes resistors, capacitors and imaginary resistors only. Because this prototype filter can be obtained by using a frequency transformation and a circuit transformation, it can be easily designed. However, in case of $(2k+1)$-th order, this filter includes $2k$ floating imaginary resistors. When a fully differential complex filter is realized by using fully differential OTA's and capacitors, a floating imaginary resistor requires twice as many OTA's as a grounded imaginary resistor. Therefore, it is desirable that the number of the floating imaginary resistors be as small as possible. Generally, a complex prototype ladder filter excluding inductors is called an $\mathrm{R^iCR}$ filter.

In this letter, a new $\mathrm{R^iCR}$ filter with finite transmission zeros is proposed. The proposed $\mathrm{R^iCR}$ filter is obtained through a frequency transformation and a new circuit transformation. When the prototype filter is realized by using OTA's and capacitors, the required active elements of the proposed ladder filter are fewer than those of the conventional one. By tuning the transconductance of all the OTA's, absolute deviation of capacitances can be compensated. The validity of the proposed method is confirmed through computer simulation and comparison of the number of the required elements.

2. Proposed Method

A floating type imaginary resistor realized by using OTA's is shown in Fig. 1. A grounded type imaginary resistor realized by using OTA's is shown in Fig. 2. From these figures, it is confirmed that a floating type imaginary resistor require twice as much OTA's as grounded type imaginary resistor. Therefore, an $\mathrm{R^iCR}$ filter includes a small number of imaginary resistance can be realized with fewer OTA's.

2.1 Cirsuit Transformation

A circuit including two imaginary resistors, a capacitor and

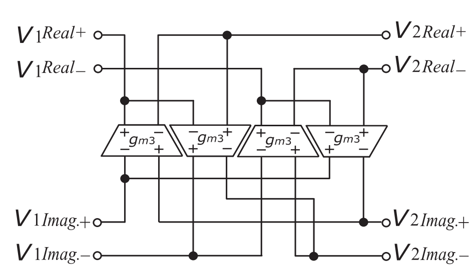

Fig. 1 Floating type imaginary resistor realized by using OTA's.

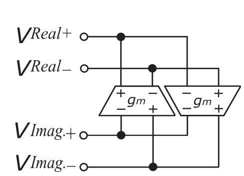

Fig. 2 Grounded type imaginary resistor realized by using OTA's.

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an inductor is shown in Fig. 3. The impedance of the circuit shown in Fig. 3 is given by

$$Z_1 = \frac{s^2 CL\left(\dfrac{R_1}{R_2}+1\right) + js\left(CR_1 - \dfrac{L}{R_2}\right) + 1}{-js^2\left(\dfrac{CL}{R_2}\right) + sC} \quad , \tag{1}$$

where j is the imaginary unit. Next, a circuit including two imaginary resistors and two capacitors is shown in Fig. 4. The impedance of the circuit shown in Fig. 4 is given by

$$Z_2 = \frac{-s^2 C_a C_b R_a R_b + js(C_a R_a + C_b R_b) + 1}{js^2 C_a C_b (R_a + R_b) + s(C_a + C_b)} \quad . \tag{2}$$

Comparing Eqs. (1) and (2) leads to

$$\left.\begin{array}{lcl} CL\left(\dfrac{R_1}{R_2}+1\right) & = & -C_a C_b R_a R_b \\ CR_1 - \dfrac{L}{R_2} & = & C_a R_a + C_b R_b \\ -\dfrac{CL}{R_2} & = & C_a C_b (R_a + R_b) \\ C & = & C_a C_b \end{array}\right\} \quad . \tag{3}$$

Solving the above equation set for the element values of the circuit shown in Fig. 4 yields

$$\left.\begin{array}{lcl} C_a & = & \dfrac{C}{2}\left(1 \mp \dfrac{\dfrac{L}{R_2} + CR_1}{\sqrt{\left(CR_1 + \dfrac{L}{R_2}\right)^2 + 4LC}}\right) \\ C_b & = & \dfrac{C}{2}\left(1 \pm \dfrac{\dfrac{L}{R_2} + CR_1}{\sqrt{\left(CR_1 + \dfrac{L}{R_2}\right)^2 + 4LC}}\right) \\ jR_a & = & j\dfrac{2(R_1+R_2)\sqrt{\left(CR_1 + \dfrac{L}{R_2}\right)^2 + 4LC}}{\left\{\begin{array}{l}\sqrt{\left(CR_1 + \dfrac{L}{R_2}\right)^2 + 4LC} \\ \mp\left(\dfrac{L}{R_2} + C(R_1 + 2R_2)\right)\end{array}\right\}} \\ jR_b & = & j\dfrac{2(R_1+R_2)\sqrt{\left(CR_1 + \dfrac{L}{R_2}\right)^2 + 4LC}}{\left\{\begin{array}{l}\sqrt{\left(CR_1 + \dfrac{L}{R_2}\right)^2 + 4LC} \\ \pm\left(\dfrac{L}{R_2} + C(R_1 + 2R_2)\right)\end{array}\right\}} \end{array}\right\} \quad . \tag{4}$$

In Eq. (4), double-sign corresponds. Therefore, the circuit including an inductor shown in Fig. 3 can be converted to the circuit including no inductors shown in Fig. 4.

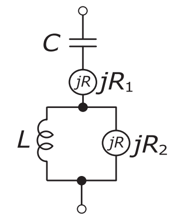

Fig. 3 A circuit including an inductor.

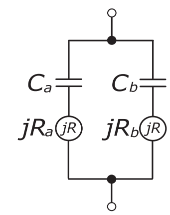

Fig. 4 Equivalent circuit of Fig. 1.

2.2 Application to a Complex Ladder Filter Obtained from a Frequency Transformation

A special case (ω_s=0) of the Extended Lowpass-Highpass Transformation (ELHT) [4] is given in the following equation.

$$x = -\frac{\omega_c}{\omega} - x_s \quad , \tag{5}$$

where ω_c and x_s are the parameters obtained from passband edges of the desired complex filter. Because the complex prototype filter should not include negative inductors and negative capacitors, $\omega_c > 0$ in general. Applying this frequency transformation to a real prototype ladder filter with finite transmission zeros, a complex ladder filter shown in Fig. 5 can be obtained. Applying the proposed circuit transformation to shunt arms of the complex filter, an R$^{\mathrm{i}}$CR filter can be obtained. It is known that the floating imaginary resistors included in the terminating circuits can be grounded using the circuit transformations [5]. Through the above circuit transformations, the circuit shown in Fig. 5 can be converted to an R$^{\mathrm{i}}$CR filter shown in Fig. 6. The element values of the shunt arms of R$^{\mathrm{i}}$CR filter become as follows:

$$\left.\begin{array}{lcl} C_{2ka} & = & \dfrac{1}{2\omega_c L_{p2k}} \\ C_{2kb} & = & \dfrac{1}{2\omega_c L_{p2k}} \\ jR_{2ka} & = & 2j\left(-x_s L_{p2k} \pm \sqrt{\dfrac{L_{p2k}}{C_{p2k}}}\right) \\ jR_{2kb} & = & 2j\left(-x_s L_{p2k} \mp \sqrt{\dfrac{L_{p2k}}{C_{p2k}}}\right) \end{array}\right\} \quad , \tag{6}$$

where C_{p2k} and L_{p2k} are the element values of the capacitors and inductors included in the shunt arms of the real prototype

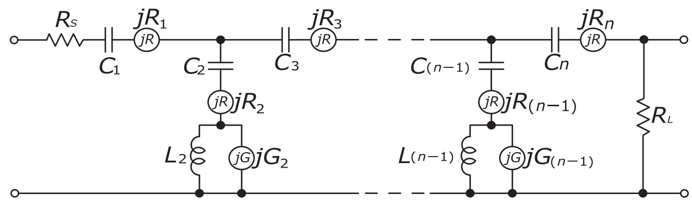

Fig. 5 Complex prototype ladder filter obtained by using ELHT method ($\omega_s = 0$).

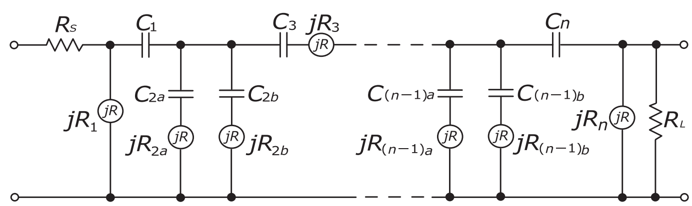

Fig. 6 Proposed prototype circuit.

ladder filter, respectively. Here, double-sign corresponds in Eq. (6). From Eq. (6), it is found that the capacitors included in the resulting circuit become positive when the inductors included in the shunt arms of the real prototype ladder filter are positive.

The element values of the series arms of R^{i}CR filter become as follows [4]:

$$\left.\begin{aligned} C_{(2k-1)} &= \frac{1}{2\omega_c L_{p(2k-1)}} \\ jR_{(2k-1)} &= -jx_s L_{p(2k-1)} \end{aligned}\right\}, \qquad (7)$$

where $L_{p(2k-1)}$ is the element values of an inductor included in a series arm of the real prototype ladder filter.

3. Design Example

A complex filter which satisfies the following specifications is designed.

Fifth-order complex elliptic filter	
Passband	3–5 rad/s
Passband ripple	1.25 dB
Minimum attenuation	60 dB

The element values of the proposed R^{i}CR filter shown in Fig. 6 are summarized in Table 1. The R^{i}CR filter is realized

Table 1 Element values of the proposed prototype circuit.

Element	Value	Element	Value
R_s	1	R_L	1.234
jR_1	$0.1266j$	jR_{2a}	$0.02936j$
jR_{2b}	$0.07411j$	jR_3	$0.1752j$
jR_{4a}	$0.006509j$	jR_{4b}	$0.03166j$
jR_5	$0.1404j$	C_1	2.140
C_{2a}	5.154	C_{2b}	5.154
C_3	1.522	C_{4a}	13.97
C_{4b}	13.97	C_5	1.924

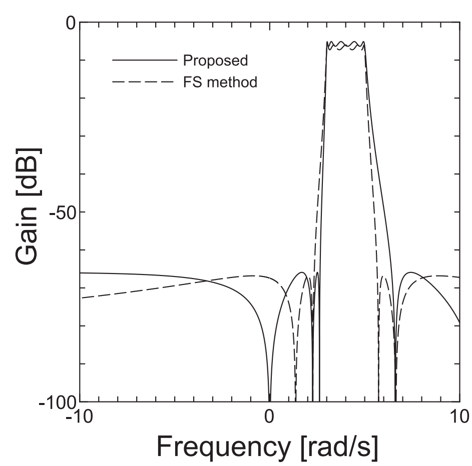

Fig. 7 Simulation result.

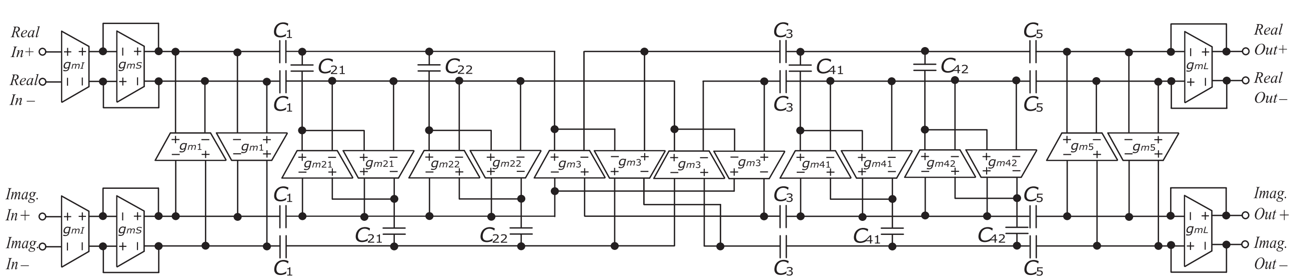

Fig. 8 Proposed circuit for $n = 5$.

Table 2 Element values of the proposed circuit.

Element	Value	Element	Value
g_{mI}	1	g_{ms}	1
g_{mL}	0.8102	g_{m1}	7.900
g_{m21}	34.06	g_{m22}	13.49
g_{m3}	11.41	g_{m41}	153.6
g_{m42}	31.59	g_{m5}	7.121
C_1	4.281	C_{21}	5.154
C_{22}	5.154	C_3	3.044
C_{41}	13.97	C_{42}	13.97
C_5	3.847		

Table 3 The number of the required components.

	Proposed		Ref. [3]		Ref. [2]	
Order	OTA	C	OTA	C	OTA	C
3rd	14	12	18	12	24	10
5th	22	20	28	22	40	18
$(2k+1)$th	$8k+6$	$8k+4$	$10k+8$	$10k+2$	$16k+8$	$8k+2$

by using ideal OTA's (VCCS's) and capacitors. The resulting circuit and its element values are shown in Fig. 8 and Table 2.

The simulation results are shown in Fig. 7. In this figure, the gain characteristics of the complex filter obtained from the Frequency Shifting (FS) method [2], [3] are also shown. From Fig. 7, it is found that the gain characteristics of the proposed circuit are duller than those of the complex filter obtained from the FS method [2], [3] in $\omega > 5$. On the other hand, the gain characteristics of the proposed circuit are sharper than that of the complex filter obtained from the FS method in $\omega < 1$. The image rejection ratio of the proposed circuit is the same as that of the conventional ones [2], [3] (60 dB). The stopband edges of the proposed circuit are $\omega = 6.4$ rad/s and $\omega = 2.6$ rad/s. Those of the conventional circuit are $\omega = 6.0$ rad/s and $\omega = 2.3$ rad/s.

In recent years, the output impedance of OTA's has been lowered due to scaledown of semiconductors. When $r_{out} \gg 1/(\omega C_{min})$ is satisfied, the degradation of the frequency response might be ignored, where r_{out} is the output resistance of an OTA and C_{min} is the minimum capacitance of the complex filter. When the passband of the proposed circuit is scaled to 3–5 MHz, $\omega = 2\pi \cdot 10^6$ and an OTA [7] (the output resistance is approximately 50 kΩ) in 0.18 μm process is employed, C_{min} should be larger than 318.3 pF in order to meet $r_{out} > 100/(\omega C_{min})$. Because the required capacitance is very large, we should increase the channel length of transistors or add the circuits like negative resistances [8] to OTA in actual.

The number of the required elements is summarized in Table 3. From this table, it is found that the required active elements of the proposed circuit are the fewest of them.

4. Conclusion

In this letter, a new R$^{\mathrm{i}}$CR filter with finite transmission zeros is proposed. The proposed circuit is obtained through a frequency transformation and a circuit transformation. The number of the required active elements of the proposed circuit is fewer than that of the conventional ones. Future works are reduction of the element value spread, realization of the proposed R$^{\mathrm{i}}$CR filter using non-ideal OTA's and design of a complex filter with variable bandwidth.

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Appendix: Conditions for a Positive Element Value

It is assumed that $C > 0$ and $L > 0$ in the prototype circuit shown in Fig. 3. From Eq. (4), it is found that $C_a > 0$ and $C_b > 0$ when the following condition is satisfied.

$$\left| \frac{\dfrac{L}{R_2} + CR_1}{\sqrt{\left(CR_1 + \dfrac{L}{R_2}\right)^2 + 4LC}} \right| \leqq 1. \tag{A$\cdot$1}$$

Organizing Eq. (A· 1) gives the following equation.

$$\left| \frac{L}{R_2} + CR_1 \right| \leqq \left| \sqrt{\left(CR_1 + \frac{L}{R_2}\right)^2 + 4LC} \right|. \tag{A$\cdot$2}$$

Squaring both sides of Eq. (A· 2) gives

$$\begin{aligned} \left(CR_1 + \frac{L}{R_2}\right)^2 &\leqq \left(\sqrt{\left(CR_1 + \frac{L}{R_2}\right)^2 + 4LC}\right)^2 . \\ 0 &\leqq 4LC \end{aligned} \tag{A$\cdot$3}$$

From the above, it is found that $C_a > 0$ and $C_b > 0$ when $C > 0$ and $L > 0$.