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Cohesive toposes of sheaves on monoids of continuous endofunctions of the unit interval.

(English) [\[Zbl 07229466\]](#)

Theory Appl. Categ. 35, 1087-1100 (2020).

Working within the context of *F. W. Lawvere* [Theory Appl. Categ. 19, 41–49 (2007; [Zbl 1123.18001](#))], *M. Menni* [Theory Appl. Categ. 29, 542–568 (2014; [Zbl 1307.18002](#))] has shown that a pre-cohesive category of presheaves over sets satisfies the continuity axiom iff it is a quality type, proceeding to construct a pre-cohesive and sufficiently cohesive topos over sets in concordance with the continuity axiom and showing that continuity and sufficient cohesion are compatible over sets. This was achieved by considering the unit interval $I = [0, 1]$, drastically cutting down the monoid of continuous endomorphisms and taking the topology of finite partitions, with respect to which the category of sheaves was taken.

This paper consists of 5 sections. §2 considers the question which is the biggest monoid of continuous endomorphisms of I admitting this construction and producing a cohesive topos over sets. It turns out that a continuous function acquiesces in the stability condition for Menni's topology iff it is unilateral (Definition 2.1). It is shown in Theorem 2.8 that Menni's topology works for any submonoid of unilateral endomorphisms of I containing the linear endomorphisms.

The construction in [loc. cit.] is to be construed as a return, against *P. T. Johnstone* [Proc. Lond. Math. Soc. (3) 38, 237–271 (1979; [Zbl 0402.18006](#))] insistence that continuous paths should be replaced by convergent sequences, to Lawvere's original idea of considering paths as the primitive notion and taking the topology so that $x \mapsto \frac{1}{2}x$ and $x \mapsto \frac{1}{2}(x + 1)$ form a cover. §3 mirrors some of the sheaf-theoretic results for the topological topos in [loc. cit.], showing that they are also valid in topological toposes in [*M. Menni*, Theory Appl. Categ. 29, 542–568 (2014; [Zbl 1307.18002](#))].

§4 constructs another topos closer to Lawvere's original idea of considering the full monoid of continuous endomorphisms of I . It is shown in 4.6 that the topos provides another model of cohesion over sets.

§5 gives a construction, simpler than that in [Johnstone, loc. cit.], that produces a presheaf failing to abide by the continuity axiom in any sufficiently cohesive topos of presheaves,

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MSC:

[18F60](#) Categories of topological spaces and continuous mappings

[18F10](#) Grothendieck topologies and Grothendieck topoi

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Full Text: [Link](#)

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