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Descotte, M. E.; **Dubuc, E. J.**; **Szyld, M.**
A localization of bicategories via homotopies. (English) Zbl 07216041
Theory Appl. Categ. 35, 845-874 (2020).

This paper is concerned with the localization of a bicategory $\mathcal{C}$, which is the process of making a family $\mathcal{W}$ of arrows of $\mathcal{C}$ into equivalences ina an appropriate universal sense [*D. A. Pronk*, Compos. Math. 102, No. 3, 243–303 (1996; Zbl 0871.18003)]. The paper deals with the situation analogous to that in [*W. G. Dwyer* et al., Homotopy limit functors on model categories and homotopical categories. Providence, RI: American Mathematical Society (AMS) (2004; Zbl 1072.18012); *M. Szyld*, "The homotopy relation in a category with weak equivalences", Preprint, `arXiv:1804.04244`], that is to say, the construction of the localization as a quotient in dimension 2, which amounts to constructing a localizing bicategory $\mathcal{C}$, resulting in a new bicategory $\mathcal{H}o\,(\mathcal{C},\mathcal{W})$ of the same objects and arrows as the original bicategory with the original 2-cells and new 2-cells corresponding to a notion of homotopy together with a natural 2-functor $i:\mathcal{C}\to\mathcal{H}o\,(\mathcal{C},\mathcal{W})$. The main result of the paper (Theorem 3.47) goes as follows:

Theorem. If $\mathcal{W}$ abides by the 3 for 2 property, and arrows of $\mathcal{W}$ can be written, up to isomorphism, as a composition of split arrows of $\mathcal{W}$, then the 2-functor $i:\mathcal{C}\to\mathcal{H}o\,(\mathcal{C},\mathcal{W})$ is the strict localization.

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MSC:

18N10 2-categories, bicategories, double categories
18N40 Homotopical algebra, Quillen model categories, derivators
18N55 Localizations (e.g., simplicial localization, Bousfield localization)

Keywords:

localization; bicategory; homotopy

Full Text: Link

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