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Network models. (English) [\[Zbl 07216038\]](#)

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This paper studies operads suitable for designing networks. The authors want a malleable framework that can handle networks of various kinds, described at a level of detail that the user is free to adjust. To achieve this malleability, the authors introduce, as a generalization of the concept of colored directed multigraphs, a general concept of *network model*, which gives an operad whose operations are ways to build larger networks of this kind from smaller ones. Formally, given a set C of vertex colors, a network model is a lax symmetric monoidal functor

$$F : \mathbf{S}(C) \rightarrow \mathbf{Cat}$$

where $\mathbf{S}(C)$ is the free strict symmetric monoidal category on C and \mathbf{Cat} is the category of small categories endowed with its cartesian monoidal structure. The main result of the paper is that any network model gives rise to a typed operad, also known as a colored operad or symmetric multicategory [D. Yau, Colored operads. Providence, RI: American Mathematical Society (AMS) (2016; [Zbl 1348.18014](#))].

By assuming that F takes values in the category \mathbf{Mon} of monoids and simultaneously that C is a singleton so that $\mathbf{S}(C)$ is the groupoid \mathbf{S} , we get what is called a one-colored network model, which is a lax symmetric monoidal functor

$$F : \mathbf{S} \rightarrow \mathbf{Mon}$$

Joyal initiated an extensive study of functors

$$F : \mathbf{S} \rightarrow \mathbf{Set}$$

under the name of *species* [F. Bergeron et al., Combinatorial species and tree-like structures. Transl. from the French by Margaret Readdy. Cambridge: Cambridge University Press (1998; [Zbl 0888.05001](#)); A. Joyal, Adv. Math. 42, 1-82 (1981; [Zbl 0491.05007](#)); Lect. Notes Math. 1234, 126-159 (1986; [Zbl 0612.18002](#))]. It can be said that a one-colored network model is a species with some extra operations.

The paper consists of 8 sections. §2 studies one-colored network models. §3 describes a systematic procedure for getting one-colored network models from monoids. §4 studies network models with examples. §5 describes a category \mathbf{NetMod} of network models, showing that the procedure for network models from monoids is functorial. Making \mathbf{NetMod} a symmetric monoidal category, the authors give examples of how to build new network models by tensoring old ones. §6 is the technical heart of the paper, providing the machinery to construct operads from network models in a functorial way and describing enhancements of the well-known Grothendieck construction of the category of elements $\int F$ of a functor $F : C \rightarrow \mathbf{Cat}$, where C is any small category. It is shown that, given a symmetric monoidal category C and a lax symmetric monoidal functor $F : C \rightarrow \mathbf{Cat}$, $\int F$ is symmetric monoidal. It is also shown that the construction sending the lax symmetric monoidal functor F to the symmetric monoidal category $\int F$ is functorial. §7 applies the machinery to build operads from network models. §8 describes some algebras of these operads, discussing an algebra whose elements are networks of range-limited communication channels (Example 8.4).

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MSC:

- 18D30 Fibered categories
- 18M05 Monoidal categories, symmetric monoidal categories
- 18M35 Categories of networks and processes, compositionality
- 18M60 Operads (general)
- 18M80 Species, Hopf monoids, operads in combinatorics
- 68M10 Network design and communication in computer systems
- 90B18 Communication networks in operations research

Keywords:

Grothendieck construction; graph; monoidal category; network; operad

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