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Multiple vector bundles: cores, splittings and decompositions. (English) Zbl 07205814

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Double vector bundles were introduced by [Esquisses Mathématiques. 29. (Directeur de la publication: Charles Ehresmann.) Pradines, Jean: Fibres vectoriels doubles et calcul des jets non holonomes. Amiens - France: M. ou Mme Ehresmann, U.E.R. de Mathematiques. (1977; Zbl 0396.53016)] in terms of double vector bundle charts for a structural tool in the study of nonholonomic jets. The modern definition of a double vector bundle was given in [*K. C. H. Mackenzie*, Adv. Math. 94, No. 2, 180–239 (1992; Zbl 0765.57025)]. It is easy to see that double vector bundles in the sense of Pradines are ones in the modern sense, but the converse, which is the existence of local double vector bundle charts, or equivalently, of local linear splittings, is by no means trivial. The first elementary construction of local splittings was given by Fernando del Carpio-Marek [`https://impa.br/wp-content/uploads/2017/05/Fernando_Del_Carpio.pdf`] under the assumption that the double projection

$$\left(p_A^D, p_B^D\right) : D \to A \times_M B$$

of a double vector bundle is a surjective submersion. The principal objective in this paper is to exploit del Carpio-Marek's method to construct local splittings of triple vector bundles.

§2 defines multiple vector bundles.§3 defines linear splittings and decompositions of n-fold vector bundles, which are shown to be equivalent (§3.1). The existence of local splittings of a given n-fold vector bundle is established in §3.4. The existence of global decompositions of n-fold vector bundles is given, and how n-fold vector bundles can alternatively be defined as smooth manifolds with an atlas of compatible n-fold vector bundle charts is explained in §3.8.

§4 demonstrates that each ∞-fold vector bundle admits a linear decompostion. §5 explains most of the constuctions and results in the paper in the special case of a triple vector bundle.

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MSC:

53C05 Connections, general theory
18F15 Abstract manifolds and fiber bundles (category-theoretic aspects)
55R65 Generalizations of fiber spaces and bundles in algebraic topology

Keywords:

n-fold vector bundle atlas; linear decomposition

Full Text: Link

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