

Berndtsson, Bo**Superforms, supercurrents, minimal manifolds and Riemannian geometry.** (English)**Zbl 07194725**[Arnold Math. J. 5, No. 4, 501-532 \(2019\).](#)

Superforms and supercurrents were introduced in [A. Lagerberg, Math. Z. 270, No. 3–4, 1011–1050 (2012; Zbl 1318.32040)] so as to study tropical varieties. This paper extends the superformalism in a different direction by associating to any smooth submanifold $M \subseteq \mathbb{R}^n$ a supercurrent $[M]_S$ in order to apply methods from complex analysis to real manifolds. It turns out that M is minimal if and only if

$$[M]_S \wedge \beta^{m-1}/(m-1)!$$

is closed, where m is the dimension of M and β is the Euclidean Kähler form on \mathbb{C}^n (Corollary 5.2), showing that minimality is characterized by a quite simple linear equation and therefore suggesting a generalization of minimal manifolds to minimal supercurrents. This is similar to the use of currents and varifolds in the theory of minimal manifolds, but, as in complex analysis, has the extra feature of a bidegree. With this in mind, the author imitates Lelong's method for positive closed currents [P. Lelong, Fonctions plurisousharmoniques et formes différentielles positives. Cours et documents de mathématiques et de physique. Paris-London-New York: Gordon & Breach distribue par Dunod éditeur (1968; Zbl 0195.11603)] so as to establish, e.g., the monotonicity formula for minimal manifolds and a volume estimate generalizing a recent result of S. Brendle and P.-K. Hung [Geom. Funct. Anal. 27, No. 2, 235–239 (2017; Zbl 1368.53006)] (Theorem 6.3). A result on removable singularities for minimal manifolds along the lines of the El Mir-Skoda theorem [H. El Mir, Acta Math. 153, 1–45 (1984; Zbl 0557.32003); H. Skoda, Invent. Math. 66, 361–376 (1982; Zbl 0488.58002)] from complex analysis (Theorem 7.2) and a formula for the variation of the volume under the mean curvature flow (Theorem 8.2) are also obtained. The author gives an expression of the Riemann curvature tensor of M as a superform, which is no other than a mere rewriting of Gauss's formula and which is applied in the last section to get a quite short proof of Weyl's tube theorem [H. Weyl, Math. Z. 12, 154–160 (1922; JFM 48.0845.01)].

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MSC:

- 58A50 Supermanifolds and graded manifolds
- 53C42 Differential geometry of immersions (minimal, prescribed curvature, tight, etc.)
- 53C15 General geometric structures on manifolds (almost complex, almost product structures, etc.)
- 49Q05 Minimal surfaces and optimization
- 14T20 Geometric aspects of tropical varieties

Full Text: [DOI](#)**References:**

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