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$\overline{\operatorname{Spec}\mathbb{Z}}$ **and the Gromov norm.** (English) Zbl 07178542

Theory Appl. Categ. 35, 155-178 (2020).

The principal objective in this paper is to use Γ-spaces [*G. Segal*, Topology 13, 293–312 (1974; Zbl 0284.55016)] so as to perform homological algebra in the category $\mathfrak{s}$-Mod by applying an analogue of Dold-Kan correspondence.

The paper consists of four sections. §2 aims to reach a good definition of the homology of a pointed simplicial set with coefficients in an $\mathfrak{s}$-module, showing that it generalizes the standard notion in algebraic topology, which is achieved in Definition 2.15 and Theorem 2.17.

§3 shows that the singular homology $H_*(X,\mathbb{R})$ of a topological space inherits a natural semi-norm from the filtration of the $\mathfrak{s}$-module $H\mathbb{R}$ by the sub-$\mathfrak{s}$-module $\|H\mathbb{R}\|_\lambda$ $(\lambda \in \mathbb{R}_+)$ associated to the archimedean place of $\overline{\operatorname{Spec}\mathbb{Z}}$ as constructed in [*A. Connes* and *C. Consani*, J. Number Theory 162, 518–551 (2016; Zbl 1409.14046)], this semi-norm being equivalent to the Gromov semi-norm on singular homology.

§4 is entirely devoted to showing that the two norms on $H_*(X,\mathbb{R})$ are equal as far as compact Riemann surfaces are concerned.

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**MSC:**

16Y60 Semirings
20N20 Hypergroups
18G55 Nonabelian homotopical algebra (MSC2010)
18G30 Simplicial sets; simplicial objects in a category (MSC2010)
18G35 Chain complexes (category-theoretic aspects), dg categories
18G60 Other (co)homology theories (MSC2010)
14G40 Arithmetic varieties and schemes; Arakelov theory; heights

**Keywords:**

gamma spaces; gamma rings; site; Gromov norm; Arakelov geometry; homology theory

**Full Text:** Link

**References:**

[1] M. Artin, A. Grothendieck, J-L. Verdier, eds. (1972), SGA4, LNM 269-270-305, Berlin; New York: Springer-Verlag.

[2] A. Connes, C. Consani,Absolute algebra and Segal's Gamma sets, J. Number Theory 162 (2016), 518-551. · Zbl 1409.14046

[3] B. Dundas, T. G. Goodwillie, R. McCarthy,The local structure of algebraic K-theory. Algebra and Applications, 18. Springer-Verlag London, Ltd., London, 2013. · Zbl 1272.55002

[4] P. G. Goerss, J. F. Jardine,Simplicial homotopy theory. Progress in Mathematics, 174. Birkhauser Verlag, Basel, 1999. · Zbl 0949.55001

[5] M. Gromov,Volume and bounded cohomology, Inst. Hautes Etudes Sci. Publ. Math. (1982), no. 56, 5-99 (1983).

[6] L. Illusie,Complexe cotangent et d´eformations. Lecture Notes in Mathematics, Vol. 283. Springer-Verlag, Berlin-New York, 1972. · Zbl 0238.13017

[7] M. Kapranov,A. Smirnov,Cohomology determinants and Reciprocity Laws (preprint).

[8] S. MacLane, I. Moerdijk,Sheaves in geometry and logic. A first introduction to topos theory. Corrected reprint of the 1992 edition. Universitext. Springer-Verlag, New York, 1994.

[9] S. MacLane,Categories for the working mathematician. Graduate Texts in Mathematics, Vol. 5. Springer-Verlag, New York-Berlin, 1971. ix+262 pp. · Zbl 0705.18001

[10] P. MaySimplicial objects in algebraic topology. Van Nostrand Mathematical Studies, No. 11 D. Van Nostrand Co., Inc., Princeton, N.J.-Toronto, Ont.-London 1967.

[11] J. C. MooreSeminar on algebraic homotopy theory, Princeton, 1956 (mimeographed notes).

[12] G. Segal,Categories and cohomology theories. Topology, 13 (1974), 293-312. · Zbl 0284.55016

[13] W. Thurston,Geometry and topology of3-manifolds, Lecture Notes Princeton (1978).