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Augmented virtual double categories. (English) [Zbl 07178546]
Theory Appl. Categ. 35, 261-325 (2020).

A. Burroni [Cah. Topologie Géom. Différ. Catégoriques 12, 215–321 (1971; Zbl 0246.18007)] generalized the notion of double category to that of *virtual double category* [*G. S. H. Cruttwell* and *M. A. Shulman*, Theory Appl. Categ. 24, 580–655 (2010; Zbl 1220.18003)], though Burroni himself called it multicatégory, in which cells have a horizontal multi-source and single horizontal target.

This paper introduces the notion of *augmented virtual double category* as a generalization of virtual double category by admitting cells to be of empty horizontal targets. The principal objective in introducing augmented virtual double categories is to internalize the notion of *Yoneda embedding*, just as Cruttwell and Shulman [loc. cit.] have internalized the notion of fully faithful morphism in the unital virtual double category $\mathbb{Mod}(\mathbb{X})$ of modules in a virtual double category $\mathbb{X}$ as well as *E. Riehl* and *D. Verity* [Algebr. Geom. Topol. 17, No. 1, 189–271 (2017; Zbl 1362.18020)] have internalized the notions of fully faithful morphism, exact square and pointwise Kan extension in the unital virtual double category $\underline{\mathrm{Mod}}_{\mathcal{K}}$ of modules between ∞-categories in the homotopy 2-category of a ∞-cosmos $\mathcal{K}$.

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MSC:

18N10   2-categories, bicategories, double categories

Keywords:

augmented virtual double category; multicategory; Yoneda structure

Full Text: Link

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