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Symmetric monoidal categories and Γ-categories. (English) Zbl 07194061

Theory Appl. Categ. 35, 417-512 (2020).

A. K. Bousfield and *E. M. Friedlander* [Lect. Notes Math. 658, 80–130 (1978; Zbl 0405.55021)] constructed a model category of Γ-spaces, establishing that its homotopy category is equivalent to the homotopy category of connective spectra, which was taken further by *S. Schwede* [Math. Proc. Camb. Philos. Soc. 126, No. 2, 329–356 (1999; Zbl 0920.55011)] to construct a Quillen equivalent model structure on Γ-spaces with fibrant objects being described as (pointed) spaces of a *coherently commutative group* structure. His model category is a symmetric monoidal closed model category under the *smash product* [*M. Lydakis*, Math. Proc. Camb. Philos. Soc. 126, No. 2, 311–328 (1999; Zbl 0996.55020)] which is just a version of the *Day convolution product* [*B. Day*, Lect. Notes Math. 137, 1–38 (1970; Zbl 0203.31402); Lect. Notes Math. 420, 20–54 (1974; Zbl 0367.18008)] for normalized functors.

This paper is the first in a series of papers on study of *coherently commutative monoidal* objects in cartesian closed model categories, whose ultimate aim is to understand coherently commutative monoidal objects in suitable model categories of (∞, n)-categories [*C. Rezk*, Geom. Topol. 14, No. 1, 521–571 (2010; Zbl 1203.18015); *D. Ara*, J. K-Theory 14, No. 3, 701–749 (2014; Zbl 1322.18002)]. This paper deals with the case of ordinary categories with a view towards the aforementioned ultimate goal.

A Γ-category is a functor from the skeletal category of finite based sets Γ^{op} into the category of all (small) categories **Cat**, Γ**Cat** denoting the category of all Γ-categories and natural transformations between them. A symmetric monoidal closed model category structure is constructed on Γ**Cat** along the lines of the construction of the stable Q-model category [*S. Schwede*, Math. Proc. Camb. Philos. Soc. 126, No. 2, 329–356 (1999; Zbl 0920.55011)]. The principal objective in this paper is to compare the category of all (small) symmetric monoidal categories with the model category of coherently commutative monoidal categories. There are many variants of the category of symmetric monoidal categories, all of which are of equivalent homotopy categories [*M. A. Mandell*, Doc. Math. 15, 765–791 (2010; Zbl 1227.19003), Theorem 3.9] but are merely fibration categories. In order to get a model category structure, the author chooses to work in a subcategory **Perm**, the objects of which are *permutative categories* (strict symmetric monoidal categories) and the morphisms of which are strict symmetric monoidal functors, and which inherits a model category structure from the natural model category structure on **Cat**, the inherited model category structure being called the *natural model category structure* of permutative categories. It follows from [*J. Bourke*, J. Homotopy Relat. Struct. 12, No. 1, 31–81 (2017; Zbl 1417.18001), Proposition 6.4] that there is a symmetric skew monoidal structure on **Perm** inducing a symmetric monoidal closed structure on the homotopy category of **Perm**.

The model category structure of *coherently commutative monoidal categories* on Γ**Cat** is obtained by localizing the projective (or strict) model category structure on Γ**Cat**, with fibrant objects being coherently commutative monoidal categories so that it deserves its name. It is shown that Γ**Cat** is a symmetric monoidal closed model category with respect to the Day convolution product. Furthermore, a pair of functors giving rise to a Quillen equivalence is explicitly described in the case of ordinary categoreis.

G. Segal [Topology 13, 293–312 (1974; Zbl 0284.55016)] described a functor from (small) symmetric monoidal categories to the category of infinite loop spaces, which factors into the *Segal's nerve functor* followed by a group completion functor. The main result of this paper is that the unnormalized Segal's nerve functor $\mathcal{K}$, an unnormalized version of Segal's nerve functor, is the right adjoint Quillen functor of a Quillen equivalence between the natural model category of permutative categories and the model category of coherently commutative monoidal categories.

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MSC:

18M05 Monoidal categories, symmetric monoidal categories
18M60 Operads (general)
18N55 Localizations (e.g., simplicial localization, Bousfield localization)
18F25 Algebraic K-theory and L-theory (category-theoretic aspects)
55P42 Stable homotopy theory, spectra
19D23 Symmetric monoidal categories

Keywords:

Segal's nerve functor; theory of bicycles; Leinster construction

Full Text: Link

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