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Orbifolds of Reshetikhin-Turaev TQFTs. (English) Zbl 07205810

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Given any modular tensor category $\mathcal{C}$, Reshetikhin and Turaev [Zbl 0725.57007; Zbl 1346.57002; Zbl 1213.57002; Zbl 0812.57003] constructed a 3-dimensional topological quantum field theory

$$\mathcal{Z}^{\mathrm{RT},\mathcal{C}}:\widehat{\mathrm{Bord}}_3\rightarrow\mathrm{vect}$$

which is intimately related to the connection between the representation theory of quantum groups and knots on the one hand and rational conformal field theory [Zbl 1417.81016] on the other. The symmetric monoidal functor $\mathcal{Z}^{\mathrm{RT},\mathcal{C}}$ acts on diffeomorphism classes of bordisms with embedded ribbons labelled with data from $\mathcal{C}$, assigning topological invariants to ribbon embeddings into 3-manifolds. The authors [Zbl 1427.81153] extended this by constructing a Reshetikhin-Turaev defect TQFT

$$\mathcal{Z}^{\mathrm{RT},\mathcal{C}}:\widehat{\mathrm{Bord}}_3^{\mathrm{def}}\left(\mathbb{D}^{\mathcal{C}}\right)\rightarrow\mathrm{vect}$$

assigning invariants to equivalence classes of stratified bordisms whose 3-, 2- and 1-strata are respectively labelled by $\mathcal{C}$, certain Frobenius algebras in $\mathcal{C}$ and their cyclic modules. Defects in Reshetikhin-Turaev theory have previously been investigated in [Zbl 1248.81206; Zbl 1269.81169; Zbl 07173974].

The principal objective in this paper is to construct orbifolds of Reshetikhin-Turaev TQFTs by using the language of defect TQFT [Zbl 07056054], which generalized the orbifold theory for 2-dimensional TQFTs in [Zbl 1360.18007] to arbitrary dimensions. As far as dimension $n=2$ is concerned, orbifold data turn out to be certain Frobenius algebras in the 2-category associated to $\mathcal{Z}$, subsuming state sum models [Zbl 1272.57023] and ordinary group orbifolds [Zbl 1311.81207; Zbl 1360.18007]. As far as general 3-dimensional defect TQFTs are concerned, the defining conditions on orbifolds data were worked out in [Zbl 07056054]. This paper, focusing on Reshetikhin-Turaev defect TQFTs $\mathcal{Z}^{\mathcal{C}}$, reformulates their orbifold conditions internally to the modular tensor category $\mathcal{C}$ (Proposition 3.5), which is key technical result to be used in the proofs of the two main results of this paper.

The first main result of the paper (Proposition 4.4 and Theorem 4.8) goes as follows:

Theorem. For every spherical fusion category $\mathcal{S}$ there is an orbifold datum $\mathcal{A}^{\mathcal{S}}$ for $\mathcal{Z}^{\mathrm{triv}}$ with

$$\mathcal{Z}^{\mathrm{triv}}_{\mathcal{A}^{\mathcal{S}}}\simeq\mathcal{Z}^{\mathrm{TV},\mathcal{S}}$$

The second main result of the paper (Theorem 5.1) goes as follows:

Theorem. Let $\mathcal{B}$ be a ribbon fusion category and let G be a finite group. Every ribbon crossed G-category

$$\widehat{\mathcal{B}}=\bigoplus_{g\in G}\mathcal{B}_g$$

such that the component $\mathcal{B}_1$ labelled by the unit $1\in G$ gives rise to an orbifold datum for $\mathcal{B}$.

Taken together, the above two theorems imply that orbifolds unify state sum models and group actions in three dimensions. As a third source of orbifold data for the Reshetikhin-Turaev defect TQFT $\mathcal{Z}^{\mathcal{C}}$, commutative Δ-separable Frobenius algebras in $\mathcal{C}$ are identified in §3.18.

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MSC:

57K16 Finite-type and quantum invariants, topological quantum field theories (TQFT)
18M20 Fusion categories, modular tensor categories, modular functors
57R56 Topological quantum field theories (aspects of differential topology)

Keywords:

topological quantum field theory; orbifold construction; Reshetikhin-Turaev theory; modular tensor categories

Full Text: Link

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