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On support varieties and tensor products for finite dimensional algebras. (English)

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J. Algebra 547, 226-237 (2020).

Carlson [Zbl 0489.20008, Zbl 0526.20040] introduced cohomological support varieties of modules over group algebras of finite groups by using the maximal ideal spectrum of the group cohomology ring. It was shown in [*G. S. Avrunin* and *L. L. Scott*, Invent. Math. 66, 277–286 (1982; Zbl 0489.20042)] that the variety of a tensor product of modules equals the intersection of those of the modules (*tensor product property*).

A theory of support varieties for arbitrary finite-dimensional algebras A was developed by using Hochschild cohomology rings [*Ø. Solberg*, Contemp. Math. 406, 239–270 (2006; Zbl 1115.16007); *N. Snashall* and *Ø. Solberg*, Proc. Lond. Math. Soc. (3) 88, No. 3, 705–732 (2004; Zbl 1067.16010); *K. Erdmann* et al., K-Theory 33, No. 1, 67–87 (2004; Zbl 1116.16007)]. Although there is in general no natural tensor product between one-sided modules, one can tensor any left A-module with a bimodule. It has therefore been asked whether we have an equality

$$V(B \otimes_A M) = V(B) \cap V(M)$$

for a bimodule B and a left A-module with a finite-dimensional algebra A. The main result of this paper (Theorem 2.2) is that when A is a quantum complete intersection of a certain type, then there exists a left A-module M and a bimodule B with

$$V(B \otimes_A M) \nsubseteq V(M)$$

meaning that the tensor product property never holds in general, whatever bimodule version of support variety theory one uses.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

18D10 Monoidal, symmetric monoidal and braided categories (MSC2010)
16D20 Bimodules in associative algebras
16E40 (Co)homology of rings and associative algebras (e.g., Hochschild, cyclic, dihedral, etc.)
16S80 Deformations of associative rings
16T05 Hopf algebras and their applications
18E30 Derived categories, triangulated categories (MSC2010)
81R50 Quantum groups and related algebraic methods applied to problems in quantum theory

Keywords:

support varieties; tensor products; quantum complete intersections

Full Text: DOI arXiv

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