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**Modern mathematics in which the ancient capital revolves. People who follow Kyoto University's Research Institute for Mathematical Sciences.** (Japanese) [\[Zbl 06912640\]](#)

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This is the unique book about the Research Institute for Mathematical Sciences (RIMS) at Kyoto University, consisting of 8 chapters from Chapter 0 to Chapter  $\infty$ .

Chapter 0 is abundant in interesting episodes on how RIMS was finally established.

Chapter 1, consisting of three sections together with one column, is concerned with so-called *Sato school*. §1 addresses the birth of *algebraic analysis*, characters on the scenes being Mikio Sato, Hikosaburo Komatsu, Kosaku Yosida, Mitsuo Morimoto, Masaki Kashiwara, Takahiro Kawai.

§2 explains how the theory of *Sato hyperfunctions* emerged. The column inserted between §2 and §3 discusses the relationship between Sato hyperfunctions and numerical analysis, speaking in terms of characters, the interaction between Mikio Sato, Hidetoshi Takahashi (a leading figure in the production of the prototype parametron-based computers PC-1 in Japan in the 1950s) and Masatake Mori [[Zbl 0225.65026](#); [Zbl 0267.65016](#)]. §3 deals with mathematical physics of Sato school, which was driven energetically by the second generation of Sato school. The major figures in the first generation of Sato school are Masaki Kashiwara and Takahiro Kawai, whose achievement has culminated in [[Zbl 0277.46039](#)]. The major figures in the second generation of Sato school are Michio Jimbo and Tetsuji Miwa, the former having become a graduate student under the direction of Sato at RIMS in 1974 and the latter having become an assistant professor at RIMS in 1973. The first work of Sato school in mathematical physics is concerned with so-called Ising model, which has resulted, in 1977, in what is called *holonomic quantum fields*. Sato believes firmly that there should be some beautiful mathematical structure behind any class of exactly soluble differential equations. The second work of Sato school in mathematical physics is concerned with soliton equations, which turned out to be no other than Plücker relations and whose solutions turned out to form an infinite-dimensional Grassmannian manifolds [[Zbl 0688.58016](#); [Zbl 0528.58020](#); [Zbl 0507.58029](#)]. The chapter is concluded with a chronology of Sato school.

Chapter 2, consisting of two sections and two columns, is concerned with *algebraic geometry*. §1 depicts Heisuke Hironaka and his work on *resolutions on singularities of algebraic varieties* [[Zbl 1420.14031](#); [Zbl 0122.38603](#)], characters on the scene being Yasuo Akizuki, David Mumford, Michael Artin, Oscar Zariski and Alexander Grothendieck.

§2 addresses Shigefumi Mori and his work on the *classification of algebraic three-folds* [[Zbl 0773.14004](#); [Zbl 1103.14301](#); [Zbl 0926.14003](#); [Zbl 1001.00023](#)], characters on the scene being Koji Doi, Masayoshi Nagata, Masaki Maruyama, Hideyasu Sumihiro, Miles Reid, Teruhisa Matsusaka and János Kollár. The first column is an introduction to algebraic geometry. The second column, concerned with algebraic geometry at Kyoto University, begins with Masazo Sono (1886–1969) [[JFM 46.0187.01](#); [JFM 47.0891.04](#)]. After the second war, Yasuo Akizuki [[Zbl 0005.38701](#); [Zbl 0005.00701](#); [Zbl 0012.24502](#); [Zbl 0012.24501](#)], having become a professor at Kyoto University in 1948, established a platform of collecting mathematical talent in Kyoto. One of the collected talent was Jun-ichi Igusa [[Zbl 0045.15803](#); [Zbl 0045.32501](#); [Zbl 0045.32502](#); [Zbl 0054.06404](#)] who served Kyoto University from 1949 to 1953 when he left for Zariski in Harvard. Akizuki learned Hodge theory from Igusa to write a book on it. The Akizuki school in the 1950s included Masayoshi Nagata [[Zbl 0045.00603](#); [Zbl 0039.26303](#); [Zbl 0042.02901](#); [Zbl 0045.16003](#); [Zbl 0054.01802](#)], Kotaro Okugawa [[Zbl 0053.02001](#); [Zbl 0056.26404](#)], Shigeo Nakano [[Zbl 0058.17202](#); [Zbl 0059.14701](#); [Zbl 0068.34403](#); [Zbl 0068.34501](#)], Yoshikazu Nakai [[Zbl 0079.36901](#); [Zbl 0079.36803](#); [Zbl 0066.14602](#); [Zbl 0077.34304](#)], Teruhisa Matsusaka [[Zbl 0045.42101](#); [Zbl 0045.24201](#); [Zbl 0045.42102](#)], Yoshiro Mori [[Zbl 0068.26304](#); [Zbl 0053.21701](#)], Mieo Nishi [[Zbl 0064.26508](#); [Zbl 0068.14801](#); [Zbl 0068.34504](#)] and Hideyuki Matsumura [[Zbl 0055.26606](#); [Zbl 0079.36802](#); [Zbl 0079.36804](#)]. Hironaka studied algebraic geometry, as a college student or a graduate student, under the direction of Yasuo Akizuki. When Shigefumi Mori was in the second year of college, Professor Doi advised him to read Weil's trilogy on algebraic geometry [[Zbl 0063.08198](#), [Zbl 0037.16202](#), [Zbl 0036.16001](#)], which he had read through for half a year. Since Doi became too busy to take care of Mori, Mori continued to study algebraic geometry under the direction

of Nagata, settling a question posed by Nagata to write a paper “On automorphisms of affine planes” as a 4th year university student, which was published in a *Kōkyūroku* of RIMS. The chapter is concluded with chronologies of Heisuke Hironaka and Shigefumi Mori.

Chapter 3, consisting of three sections together with one column, is concerned with *number theory*. RIMS started without any number theorist, and it was in 1989, when RIMS had already passed 26 years since its foundation, and the incumbent director of RIMS at that time was Mikio Sato, who is versed not only in algebraic analysis but also in number theory, that a number theorist called Yasutaka Ihara arrived at RIMS from the University of Tokyo.

§1 addresses the classics of Japan’s number theory, namely, Teiji Takagi and his class field theory [JFM 47.0147.03; JFM 48.0169.01; Zbl 0176.33504]. Class field theory is at the bottom of Japan’s number theory, and the University of Tokyo was its stronghold whose first generation was Masao Sugawara [Zbl 0013.19602; Zbl 0013.38902], Shokichi Iyanaga [JFM 55.0103.07] and Yuki Yoshi Kawada [Zbl 0019.24704], and whose second generation was Goro Shimura [Zbl 0142.05402; Zbl 0141.38704], Yutaka Taniyama [Zbl 0213.22803; Zbl 0090.25703] and Kenkichi Iwasawa [Zbl 0090.02903; Zbl 0093.04403; Zbl 0202.33102]. In the international symposium on algebraic number theory [Zbl 0071.26501], André Weil met Taniyama and Shimura. All the three arrived almost simultaneously at the idea of algebra-geometric treatment of *complex multiplication*.

§2 is concerned with Yasutaka Ihara and his *nonabelian class field theory* [Zbl 0231.12017], which has greatly influenced the development of the so-called Langlands program [Zbl 0225.14022; Zbl 0225.14023] and Pierre Deligne in France, and which is indeed a rainbow bridge between the Riemann zeta function and the Selberg zeta function. In the 1980s Ihara was interested in action of Galois groups on fundamental groups [Zbl 0757.20007]. After the arrival of Ihara at RIMS, Takayuki Oda [Zbl 0718.11021; Zbl 0958.11037; Zbl 0812.11033] and Makoto Matsumoto [Zbl 0900.14001; Zbl 0867.14011; Zbl 0858.12002] joined the number-theoretic group at RIMS, which was followed by Akio Tamagawa [Zbl 1206.11081; Zbl 1194.14044], Takeshi Tsuji [Zbl 1423.14147; Zbl 1342.14045] and Shinichi Mochizuki. The Grothendieck conjecture on anabelian geometry was partially solved by Hiroaki Nakamura and Akio Tamagawa, and was finally solved completely by Shinichi Mochizuki [Zbl 0943.14014] exploiting his  $p$ -adic Hodge theory [Zbl 1091.14501]. Ihara, who prefers the delicate music of Wolfgang Amadeus Mozart, depicted the work of the three as Wagner-like, saying that the framework is grandiose while *anabelian geometry* seems to beat the air. In March 2002 Ihara retired from RIMS.

§3 is concerned with Mochizuki’s *inter-universal Teichmüller theory* [Zbl 1403.14061] and his alleged solution on the *ABC conjecture*. Here I refrain from doing more than only giving a sincere report of Peter Scholze and Jacob Stix on Mochizuki’s work [<http://www.kurims.kyoto-u.ac.jp/~motizuki/SS2018-08.pdf>] and an article in *Nature* [<https://www.nature.com/articles/d41586-020-00998-2>]. Then comes a column on number theory of Mikio Sato. In 1960s Sato was actively engaged in number theory. It was the Ramanujan conjecture that Sato first tackled in number theory. This led Sato to the so-called *Sato-Tate conjecture* [Zbl 0213.22804], which was settled by Richard Taylor and others [Zbl 1169.11021; Zbl 1169.11020; Zbl 1263.11061; Zbl 1264.11044]. To find out a general theory of Zeta functions and their variants occurring in various arenas such as Dirichlet series, Sato invented a theory of prehomogeneous vector spaces, to which Tatsuo Kimura contributed much, finally publishing a book on it [Zbl 1035.11060]. Masaki Kashiwara did computation in prehomogeneous vector spaces by using algebraic analysis [Zbl 0381.43005; Zbl 0456.58034]. The chapter is closed with a chronology of number theory beginning with the publication of Teiji Takagi’s [JFM 34.0237.01] in 1903 and ending with Mochizuki’s alleged announcement of his solution of the so-called *ABC conjecture* in 2012.

Chapter 4, consisting of three sections, is concerned with mathematical physics. §1 addresses Huzihiro Araki and the *theory of operator algebras*, characters on the scene being Huzihiro Araki, Alain Connes, Daniel Kastler, Rudolf Haag, Minoru Tomita, Masamichi Takesaki and Robert Powers. Araki became a professor of RIMS in 1964 and retired in 1997.

§2 discusses Noboru Nakanishi and his quantum field theory. In 1955 Nakanishi as well as Huzihiro Araki entered the graduate school at Kyoto University to study physics under the direction of Hideki Yukawa, a Nobel prize laureate. Nakanishi made his debut in the community of physicists by his *topological formula of Feynman integrals* [Zbl 0077.21603], claiming that Feynman integrals are determined completely by the topology of Feynman diagrams. He has finally published a book [Noboru Nakanishi, *Graph Theory and Feynman Integrals*, Gordon & Breach Science Pub., 1971]. Masaki Kashiwara and others addressed Feynman integrals from the standpoint of algebraic analysis in the 1970s [Zbl 0454.46034; Zbl 0392.46025; Zbl 0449.35095; Zbl 0527.58036; Zbl 0454.46035; Zbl 0385.35003]. During

1963–1965 Nakanishi stayed at Brookhaven National Laboratory, when he studied analyticity of scattering amplitudes and Bethe-Salpeter equations, which resulted in so-called *Nakanishi-Lautrup formalism of abelian gauge theory* [Noboru Nakanishi, “Ordinary and generalized Bethe-Salpeter equations in the unequal-mass case”, Phys. Rev. 147, 1153 (1966); Noboru Nakanishi, “Covariant quantization of the electromagnetic field in the Landau gauge”, Progress of Theoretical Physics 35, 1111–1116 (1966); Noboru Nakanishi, “Quantum electrodynamics in the general covariant gauge”, Progress of Theoretical Physics 38, 881–891 (1967)]. Nakanishi became a member of RIMS in 1966 and retired in 1997. In 1977, after Nakanishi-Lautrup formalism Taichiro Kugo and Izumi Ojima succeeded in canonically quantizing gauge fields [Zbl 1098.81591; Zbl 1098.81592]. Ojima studied mathematical physics under the direction of both Huzihiro Araki and Noboru Nakanishi after he graduated from the Faculty of Medicine at Kyoto University. It became Nakanishi’s life work to quantize gravity after Nakanishi-Lautrup formalism [Zbl 1098.83558; Zbl 1098.83515; Zbl 1098.83517; Zbl 1098.83518; Zbl 1098.83561; Zbl 1074.83513; Zbl 1098.83559; Zbl 1098.83514; Zbl 1060.83515; Zbl 1060.83516; Zbl 1059.83511; Zbl 1074.83515; Zbl 1074.83514; Zbl 1059.83513; Zbl 1059.83512; Zbl 1098.83560; Zbl 1098.83562; Zbl 0545.53055; Zbl 0527.53048; Zbl 1046.83503; Zbl 0979.83508; Zbl 0979.83506; Zbl 0672.53069; Zbl 1058.83513]. Both Nakanishi as well as Richard Feynman is strongly opposed against superstring theory, saying that superstring theory is of no coherent theory.

§3 is concerned with Hirosi Ooguri and *superstring theory*. It was Mikio Sato, the then incumbent director of RIMS, that invited Ooguri to RIMS in 1990, saying that superstring theory is mathematically attractive and affluent, to say nothing of the problem whether it is really the theory of elementary particles. Ooguri left RIMS for the University of California at Berkeley in 1994. In the 1990s Ooguri and other three arrived at *BCOV equations* [Zbl 0815.53082; Zbl 0908.58074; Zbl 0899.32008; Zbl 0919.58067], showing that topological string theory is useful in actual computation. In 2004 Ooguri and other two obtained the so-called *OSV formula* [Hirosi Ooguri, Andrew Strominger and Cumrun Vafa, “Black hole attractors and topological string”, Physical Review D (3) 70, 106007 (2004)], announcing a beautiful and highly unexpected proposal that the number of black hole states in certain string theories obtained by compactification on a Calabi-Yau manifold  $X$  is to be expressed in terms of the topological string partition function of  $X$ , namely in terms of the so-called Gromov-Witten invariants of  $X$ , which has won the Eisenbud prize for 2008. In 2010 Ooguri and others [Zbl 1266.58008] claimed that the elliptic genus of the  $K3$  surface has a natural decomposition in terms of dimensions of irreducible representations of the largest Mathieu group  $M_{24}$ , suggesting that there is a sigma-model conformal field theory with  $K3$  target of  $M_{24}$  symmetry. The chapter is concluded with chronologies of (1) operator algebras and Huzihiro Araki, (2) quantum field theory and Noboru Nakanishi, and (3) superstring theory and Hirosi Ooguri.

Chapter 5, being neither divided into sections nor accompanied by a chronology, is concerned with Kiyosi Itô and his theory of *stochastic integration* and *stochastic differential equations* known as *Itô calculus*. Since he got married young on graduation from the University of Tokyo, he worked first for the Ministry of Finance and then for the Statistics Bureau of Japan, just as young Albert Einstein worked at the Federal Office for Intellectual Property in Bern, when he found out the theory of special relativity. The basic idea of Itô calculus which he encountered while working at the Statistics Bureau of Japan for five years was published in Gully printing [Kiyosi Itô, Differential equations determining a Markov process, Zenkoku Sizyo Sugaku Danwakai-si (J. Pan-Japan Math. Coll.), 1942, 1352–1400]. It was written in Japanese and was translated into English to be published by American Mathematical Society [Zbl 0054.05803]. The development of Itô calculus can be seen in his [Zbl 0060.29105; Zbl 0063.02992; Zbl 0039.35103; Zbl 0045.07603; Zbl 0044.12202; Zbl 0049.08602; Zbl 0053.27302].

In 1943 Itô became an associate professor of the newly established Department of Mathematics at the Faculty of Science of Nagoya University, where he encountered other members Sigekatu Kuroda [Zbl 0060.08903; Zbl 0061.05901], Kiyoshi Noshiro [Zbl 0024.33002; Zbl 0021.23903], Kosaku Yosida [Zbl 0023.39702; Zbl 0024.04201; Zbl 0024.21202] and Tadasi Nakayama [Zbl 0061.04001; Zbl 0021.29402; Zbl 0019.10202].

In 1952, thanks to Yasuo Akizuki whose speciality is far away from that of Itô but who was enthusiastic over collecting excellent and challenging young mathematicians, Itô as well as Masayoshi Nagata moved from Nagoya University to Kyoto University. Itô stayed at the Institute for Advanced Study at Princeton for two years from 1954, where he got acquainted with a then graduate student Henry McKean of professor William Feller and finally published a book with him [Zbl 0285.60063]. In 1966 Itô left Japan for Aarhus University in Denmark, where he stayed until 1969 and then moved to Cornell University in USA. In 1975 Itô returned to RIMS in Japan, and was the incumbent director of RIMS from 1976 until he retired in 1979. In 1975 Itô invited Heisuke Hironaka from Harvard to RIMS. Here is a conversation between

them on that occasion, showing great flexibility of personnel affairs at RIMS.

- Itô: Now there is a vacancy on professorship at the Division of Nonlinearity at RIMS (the associate professor at that division was Takahiro Kawai and the assistant professor there was Michio Jimbo at that time).
- Hironaka: I have studied algebra, algebraic geometry and commutative algebra, where only natural and simple objects are considered. I have never used even partial differentiation in my work.
- Itô: Don't worry. Even in algebraic geometry, polynomials are used. Polynomials are undoubtedly nonlinear functions in general, aren't they? So you are indeed versed in nonlinearity.

I became a graduate student at RIMS in April 1976, for that I had to pass a two-days examination in Summer 1975, the first day being a writing examination and the second day being an interview. Here is a conversation between Itô and me (Nishimura) on that interview.

- Itô: Are there any sets which are not Lebesgue measurable?
- Nishimura: Yes, of course. By cardinality argument, ...
- Itô: No, I want a constructive proof!
- Nishimura: OK. Wait just for a moment. ...
- Itô: We have no time to wait. That is OK. As far as you are concerned, everything is good enough. Do you need a scholarship? ...

Chapter 6, consisting of three sections with a column, is concerned with applied mathematics. §1 is concerned with computer science, centering on Kyoto Common Lisp. Characters on the scene are Reiji Nakajima [Zbl 0354.02023; Zbl 0373.68025; Zbl 0432.68011; Zbl 0519.68003; Zbl 0595.68008], Taiichi Yuasa [Zbl 0442.60093; Zbl 0625.68006; Zbl 0649.68003] and Masami Hagiya [Zbl 0592.68032; Zbl 0522.03041; Zbl 0712.68057; Zbl 0873.68186]. It is regrettable that Takeshi Hayashi [Zbl 0607.68060; Zbl 0602.68036; Zbl 0544.68042; Zbl 0499.68029; Zbl 0391.03012; Zbl 0319.68043], Hirokazu Nishimura [Zbl 0401.03005; Zbl 0437.03034; Zbl 0423.68005; Zbl 1273.03089; Zbl 0574.51012], Hiroakira Ono [Zbl 0281.02033; Zbl 0253.02022; Zbl 0246.02021; Zbl 0249.02022; Zbl 0226.02025], Susumu Hayashi [Zbl 0618.54030; Zbl 0592.03010; Zbl 0513.68022; Zbl 0514.03035], Masahiko Sato [Zbl 0444.03010; Zbl 0405.03013; Zbl 0405.03013; Zbl 0274.02029], Takumi Kasai [Zbl 0301.68081; Zbl 0291.68005; Zbl 0289.68041; Zbl 0385.68047] should be missing here. All of them were once assistant professors at the division of Computer Science of RIMS.

§2 addresses fluid dynamics, in particular, *Navier-Stokes equation*. Characters on the scene are Kanefusa Gotoh [Zbl 0112.19604; Zbl 0112.19603; Zbl 0095.21702; Zbl 0227.76056; Zbl 0226.76015], Shigeo Kida [Zbl 0683.76036; Zbl 0673.76069; Zbl 0721.76041; Zbl 0712.76052; Zbl 0748.76064; Zbl 0758.76029; Zbl 1023.76557; Zbl 0939.76537; Zbl 0939.76554] and Hisashi Okamoto [Zbl 0597.35104; Zbl 0596.76119; Zbl 0558.76111; Zbl 0678.76013; Zbl 0668.35006; Zbl 0617.35134; Zbl 0925.76104; Zbl 0850.76796; Zbl 0939.76513; Zbl 1306.76016; Zbl 0991.76526].

§3 deals with *optimization*, centering upon *discrete convex analysis*. Characters on the scene are Masao Iri [Zbl 0524.94033; Zbl 0451.90053], Fujishige [Zbl 0571.90062; Zbl 0563.06010; Zbl 0665.90074; Zbl 0658.90062; Zbl 0770.90073; Zbl 0855.68107; Zbl 1296.90104; Zbl 1205.05237], Kazuo Murota [Zbl 0639.05037; Zbl 0721.73021; Zbl 0711.68066; Zbl 0834.05037; Zbl 0938.65519; Zbl 1274.90528] and Satoru Iwata [Zbl 0838.05024; Zbl 0892.65027; Zbl 1296.90103; Zbl 1102.05048; Zbl 1193.05131; Zbl 1191.94167].

Chapter  $\infty$  is concerned with the future of RIMS and that of mathematics itself. To do mathematics, we need nothing but enough time to concentrate. I became an assistant professor at RIMS in November 1979, when I enjoyed the following short conversation with Professor Satoru Takasu inviting me to assistant professorship at RIMS.

- Nishimura: What is the duty of an assistant professor at RIMS ?
- Takasu: You have only one duty, which is to get a salary every month. You should not forget your duty.

I was lucky to enjoy that position until March 1986. RIMS can afford every member but professors affluent time for mathematics.

Reviewer: Hirokazu Nishimura (Tsukuba)

**MSC:**

- 01A74 History of mathematics at institutions and academies (non-university)
- 01A60 History of mathematics in the 20th century
- 01A61 History of mathematics in the 21st century