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Picard-Vessiot extensions of Artinian simple module algebras. (English) Zbl 1107.16037

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The Picard-Vessiot theory [*M. van der Put* and *M. F. Singer*, Galois theory of linear differential equations. Berlin: Springer (2003; Zbl 1036.12008)] is a differential extension of the classical Galois theory of field extensions. *M. Takeuchi* [J. Algebra 122, No. 2, 481–509 (1989; Zbl 0669.16004)] has succeeded in pinpointing the very heart of the theory within the context of his favorite Hopf algebras. *M. van der Put* and *M. F. Singer* [Galois theory of difference equations. Berlin: Springer (1997; Zbl 0930.12006)] deals comprehensively with a difference analog of the Picard-Vessiot theory. *Y. André* [Ann. Sci. Éc. Norm. Supér. (4) 34, No. 5, 685–739 (2001; Zbl 1010.12004)] gave a unified treatment of the Picard-Vessiot theories of both differential and difference equations from a standpoint of noncommutive geometry. Enlightened by [*M. Takeuchi*, J. Algebra 122, No. 2, 481–509 (1989; Zbl 0669.16004)], this paper gives another unified approach to the Picard-Vessiot theory of equations of both types within the context of artinian simple module algebras over a cocommutative pointed smooth Hopf algebra D, in which we have

$$D = D^1 \# RG$$

with R being the base field, $G = G(D)$ being the grouplikes of D, and D^1 being the irreducible component about 1. The advantage of affine group schemes over algebraic matrix groups enabled the authors to get rid of many unessential assumptions from the Picard-Vessiot theory (e.g., the assumption that the constant field is algebraically closed).

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MSC:

16T05 Hopf algebras and their applications
12H05 Differential algebra
12H10 Difference algebra
34M55 Painlevé and other special ordinary differential equations in the complex domain; classification, hierarchies

Keywords:

pointed smooth Hopf algebras; Picard-Vessiot extensions; Artinian simple module algebras; differential fields; difference fields

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