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Liouville extensions of Artinian simple module algebras. (English) Zbl 1145.12003

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An extension of a differential field of characteristic zero is called *Liouville* provided that it contains no new constants and is obtained by iterating integrations, exponentiations and algebraic extensions. This notion first appeared in [*E. R. Kolchin*, Ann. Math. (2) 49, 1–42 (1948; Zbl 0037.18701)]. It is well known that a Picard-Vessiot extension is Liouville iff the connected component of its differential Galois group is solvable.

C. H. Franke [Trans. Am. Math. Soc. 108, 491–515 (1963; Zbl 0116.02604); Proc. Am. Math. Soc. 17, 240–246 (1966; Zbl 0143.06102); ibid. 18, 548–551 (1967; Zbl 0154.03803); Proc. Am. Math. Soc. 21, 397–401 (1969; Zbl 0177.30201)] studied the Galois correspondence for linear homogeneous difference equations in the 1960s, the first paper dealing with Liouville extensions of difference fields somewhat incompletely because of the prematurity of the Picard-Vessiot theory for difference equations. The modern treatment of the Picard-Vessiot theory for difference equations can be seen in [*M. van der Put* and *M. F. Singer*, Galois theory of difference equations. Berlin: Springer (1997; Zbl 0930.12006)]. *P. A. Hendriks* and *M. F. Singer* [J. Symb. Comput. 27, No. 3, 239–259 (1999; Zbl 0930.39004)] have established that the Galois group of a linear difference equation with rational function coefficients is solvable iff the solution space of the equation has a basis consisting of Liouvillian sequences, providing an algorithm for finding all Liouvillian solutions of a given linear difference equation.

Inspired by [*M. Takeuchi*, J. Algebra 122, No. 2, 481–509 (1989; Zbl 0669.16004)], the author and *A. Masuoka* [J. Algebra 285, No. 2, 743–767 (2005; Zbl 1107.16037)] developed a Picard-Vessiot theory for module agebras over a cocommutative pointed smooth Hopf algebra D. This paper generalizes the notion of Liouville extensions for such commutative-module algebras and characterizes the corresponding properties of affine group schemes. The characterization of Liouville Picard-Vessiot extensions in terms of affine group schemes is subtle.

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MSC:

12H05 Differential algebra

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affine group scheme; difference algebra; differential algebra; Hopf algebra; Liouville extension; Picard-Vessiot theory

Full Text: DOI

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