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Infinite-dimensional version of the Friedrichs inequality. (English. Russian original)

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The classical Friedrichs inequality is of the following form

$$\int_G u^2 d\lambda \leq C \left(\int_G \sum_{k=1}^{n} \left(\frac{\partial u}{\partial x_k} \right)^2 d\lambda + \int_S (\gamma(u))^2 d\sigma \right)$$

where

[1] G is a bounded domain in $\mathbb{R}^n$ with its boundary S abiding by certain conditions.

[2] The function u belongs to $W_2^1(G)$.

[3] λ is the classical Lebesgue measure in $\mathbb{R}^n$.

[4] σ is the surface measure on S.

[5] $\gamma : W_2^1(G) \to L_2(S)$ is the corresponding trace operator, and the constant C is specified by the geometry of the domain G.

This paper proposes two infinite-dimensional variants of the above inequality. The second variant is obtained on the basis of construction of the associated surface measure in a Hilbert space. The definition of the measure and a series of its applications are to be seen in the author's [Ukr. Math. J. 63, No. 9, 1336–1348 (2012; Zbl 1260.35021); Ukr. Math. J. 64, No. 10, 1475–1494 (2013; Zbl 1287.58005); Ukr. Math. J. 67, No. 11, 1629–1642 (2016; Zbl 1387.46030); Ukr. Math. J. 68, No. 4, 515–525 (2016; Zbl 07030359)].

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MSC:

58C	Calculus on manifolds; nonlinear operators
46T	Nonlinear functional analysis
46	Functional analysis
46F	Distributions, generalized functions, distribution spaces
46G	Measures, integration, derivative, holomorphy (all involving infinite-dimensional spaces)

Full Text: DOI

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