

**Lambek, Joachim**

**Pregroups and natural language processing.** (English) Zbl 1167.68453

Math. Intell. 28, No. 2, 41-48 (2006).

The author proposes residuated monoids (partially ordered monoids with two binary operations  $/$  and  $\backslash$  abiding by the rule

$$ab \rightarrow c \iff a \rightarrow c/b \iff b \rightarrow a \backslash c$$

as a mathematical framework for grammars of natural languages [the author, *Cat. Grammar*, 153–172 (1988; [Zbl 0692.03019](#))]. A residuated monoid with lattice operations becomes a Grishin algebra if a so-called dualizing element  $0$ , subject to the conditions

$$(0/a) \backslash 0 = a = 0 / (a \backslash 0),$$

is added, in which the de Morgan dual  $+$  to the juxtaposition standing for the monoid operation is defined by

$$a + b = ((0/b) (0/a)) \backslash 0$$

A pregroup is a Grishin algebra in which

$$0 = 1 \text{ and } a + b = ab$$

hold. A residuated monoid is a semi-Heyting algebra if it satisfies

$$a \rightarrow 1, a \rightarrow aa \text{ and } ab \rightarrow ba$$

From a logical point of view, free residuated monoids, Grishin algebras, pregroups and semi-Heyting algebras correspond to syntactic calculus, classical bilinear logic, compact bilinear logic and positive intuitionistic propositional calculus respectively.

This expository paper, which is an expanded version of the author's [Mathematics and its applications (Dordrecht) 565, 389–397 (2004; [Zbl 1078.18004](#))], explains why the author has changed his belief from [[Zbl 0692.03019](#)] to pregroups.

Reviewer: [Hirokazu Nishimura \(Tsukuba\)](#)

**MSC:**

68T50 Natural language processing

03B65 Logic of natural languages

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categoryal grammar; pregroup; residuated monoid; semi-Heyting algebra; classical bilinear logic; compact bilinear logic; positive intuitionistic propositional calculus

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