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Natural and gauge natural formalism for classical field theories. A geometric perspective including spinors and gauge theories. (English) Zbl 1138.81303

Dordrecht: Kluwer Academic Publishers (ISBN 1-4020-1703-0/hbk). xxi, 365 p. (2003).

The principal aim in this book is to show that the language of gauge natural bundles can provide a single unifying framework for all physics of fundamental interactions, in which all structural issues of Lagrangian field theories, from field equations to symmetries and conservation laws, can be discussed.

The book is divided into three parts. The first part is devoted to the geometric setting of fiber bundles with a flavor of natural bundles, assuming the reader to be familiar with such fundamental notions of differential gemetry as manifolds, Riemannian or pseudo-Riemannian structures as well as their metric connections, the Riemann tensor as well as its contractions, the Ricci tensor, the Ricci scalar, differential forms as well as their integration, etc.

The second part consists of three chapters. Chapter 6 introduces the geometric framework for the calculus of variations on fiber bundles, and defines structures concerned with the description of dynamics such as Poincaré-Cartan forms and the first variation formula, where a general viewpoint is stressed so that all the structures are described in their most general formulation without any physical restrictions or requirements. After presenting Nöther's theorem about conserved quantities in field theories, the authors analyze in detail mechanics of holonormic systems, test particles in special relativity, and Yang-Mills theories. A brief account of BRST transformations is also given. Chapter 7 discusses natural field theories as a generalization of general relativity, and investigate the consequences of naturality in terms of conserved quantities. As applications the authors consider geodestic motions on a surface, general relativity in its metric and metric-affine formulations. Chapter 8 introduces gauge natural theories as a generalization of both natural theories and pure gauge theories. Conserved quantities and superpotentials are investigated for an arbitrary gauge natural theory. Some relevant examples from physics are considered in detail.

The third part, consisting of two chapters, is an introduction to spinor fields as a genuine example of fundamental physical fields which do not admit in general a purely natural or purely gauge formulation, but can be fully understood as dynamical fields over non-trivial spacetime manifolds only on gauge natural grounds. Chapter 9 defines Clifford algebras and spin groups. The authors then introduce spin structures in their standard formulation and their relations with the Dirac operator. Spin frames are introduced and used to provide a formulation of gravity in interaction with spinor fields. Chapter 10 presents some examples and applications of spinor theories both for commuting and anticommuting spinors, say, the ordinary Dirac theory and the massless neutrino theory as well as the Wess-Zumino model which deals with Majorana anticommuting spinors.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

81-02 Research exposition (monographs, survey articles) pertaining to quantum theory

Cited in **33** Documents

Keywords:

configuration bundle; conserved quantities; Yang-Mills fields; variational morphism; vector field; spin frame; spin structure; spinors; structure bundle; tangent bundle; principal bundle; natural bundle; Nöther's theorem; Klein-Gordon Lagrangian; supersymmetry; superpotential; reference background field; Euler-Lagrangian morphism; jet bundle; Lagrangian; Hopf bundle; Jacobian; gauge potential; covariant derivative; Lie derivative; general relativity; covariant derivative; Euler-Lagrange equations; Kosmann lift; Riemann tensor; field equations; covariance; functor; curvature; fiber; canonical right action; category; affine bundle; associated bundle; base manifold; Bianchi identities; curve; gauge natural bundle; Dirac matrices; deformation; pull-back forms; orthogonal frame

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