Mangiarotti, L.; **Sardanashvily, G.**

Connections in classical and quantum field theory. (English) Zbl 1053.53022

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This book is divided into two parts, the first of them is concerned with the theory of connections in classical field theory, while the second deals with that in quantum field theory. Both parts consist of 7 chapters.

Chapter 1 summarizes basic notions on fiber bundles and jet manifolds, while Chapter 2 is devoted to the general notion of connections on fiber bundles. In Lagrangian field theory one encounters connections in relation to the following three constructions:

(1) Every second order dynamic equation on a fiber bundle $Y \to X$ is a second order holonomic connection on it.

(2) In the physically relevant case of quadratic Lagrangians, a Lagrangian L always factorizes through the covariant differential D_Γ of some connection Γ.

(3) Different connections on a fiber bundle $Y \to X$ correspond to different energy-momentum currents of fields, which differ from each other in Noether currents.

Chapter 3 is devoted to these topics. In Chapter 4, covariant Hamiltonian field theory is formulated in terms of Hamiltonian connections and Hamiltonian forms which contain the connection dependent terms. Chapter 5 is devoted to time-dependent mechanics, in which connections play a prominent role. In Chapter 6 the authors present gauge theory of principal connections as a particular case of geometric field theory formulated in terms of jet manifolds, in which the main ingredient is the bundle $C = J^1P/G$ of principal connections whose sections are principal connections on a principal bundle P with a structure group G. Chapter 7 deals with the connections in relation to gravitational theory, namely, linear and affine connections on the tangent bundle TX of a world manifold X, spinor connections, the associated principal connections on the fiber bundle LX of linear frames in TX and some others.

In quantum theory we almost never meet bundles in their traditional guises as fibrations of manifolds but in their algebraic guises as modules and their sheaves of sections. Therefore Chapter 8 is devoted to the description of jets and connections on modules and sheaves over commutative algebras. Such algebraic notions are equivalent to the corresponding geometric ones in the case of smooth vector bundles. In Chapter 9 the authors generalize their algebraic approach to the graded case (i.e., superconnections), while Chapter 14 is concerned with its noncommutative generalization. Chapter 10 gives a few examples of an application of connections to quantization of mechanical systems, namely, linear connections on Kähler manifolds in geometric quantization, symplectic connections in the Fedosov deformation quantization and Berry connections. Chapter 11 begins with the canonical connection on infinite order jets, which leads to the variational bicomplex and the algebraic approach to the calculus of variations. In the final section of this chapter it is shown that the BRST operator acting on so-called BRST tensor fields can be phrased in terms of generalized or BRST connections, in which the bicomplex extended to a graded algebra plays an significant role.

Chapter 12 is concerned with topological field theories. There are two types of topological field theories, namely, Witten type and Schwartz type. The former is exemplified by the Donaldson theory, while the latter includes models whose action functionals are independent of a metric, e.g., the Chern-Simon theory. The authors consider the truly surprising fact that the formulas of the curvature of a connection on the space of principal connections are identitical to the BRST transformations of the geometric sector of the Donaldson theory, so that the Donaldson invariants play the role of observables in topological field theory. In Chaper 13 the authors are concerned with the geometric origin of anomalies based on geometry and topology of spaces of principal connections, say, gauge anomalies, global anomalies and BRST anomalies.

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MSC:

53-02	Research exposition (monographs, survey articles) pertaining to differential geometry	Cited in **34** Documents
53C05	Connections, general theory	
53D50	Geometric quantization	
53D55	Deformation quantization, star products	
58-02	Research exposition (monographs, survey articles) pertaining to global analysis	
70Sxx	Classical field theories	
81T70	Quantization in field theory; cohomological methods	
81T75	Noncommutative geometry methods in quantum field theory	

Keywords:

spin connection; principal connection; superconnection; linear connection; affine connection; Hamiltonian connection; Berry connection; variational bicomplex; BRST connection; algebraic connection; topological field theory; Donaldson invariant; universal connection; Dubois-Violette connection; noncommutative geometry; anomalies; geometric quantization; deformation quantization; relativistic mechanics; Lagrangian field theory; Hamiltonian field theory; jet manifold; K-theory; Ne'eman-Quillen connection