Rosenthal, Kimmo I.

The theory of quantaloids. (English) Zbl 0845.18003

Pitman Research Notes in Mathematics Series. 348. Harlow: Addison Wesley Longman. 147 p. (1996).

Quantales are complete lattices endowed with a sup-preserving associative binary relation, and the author's former monograph [Quantales and their applications, Pitman Res. Notes Math. Ser. 234 (1990; Zbl 0703.06007)] serves as an indispensable handbook to active researchers related with the field and as a good introduction to novices in the field. Quantaloids, being a natural generalization of quantales, are locally small categories whose hom-sets are complete lattices with composition preserving sups in both variables, and the author's present monograph, consisting of five chapters, gives an up-to-date perspective on their art.

After giving a definition of quantaloid with examples in chapter 1, the author discusses several methods of producing new quantaloids from old ones in chapter 2. Chapter 3 is devoted to free quantaloids $\mathcal{P}(\mathcal{A})$ on locally small categories $\mathcal{A}$. Chapter 4 deals with automata and tree automata from a standpoint of enriched category theory. It is stressed that the passage from automata to tree automata is essentially the passage from a one object base quantaloid to a more general one. The last chapter discusses the general theory of modules and bimodules over quantaloids as well as its relation to the theory of *-autonomous categories.

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MSC:

18B35	Preorders, orders, domains and lattices (viewed as categories)
06B23	Complete lattices, completions
18D20	Enriched categories (over closed or monoidal categories)
03G30	Categorical logic, topoi
68Q70	Algebraic theory of languages and automata
18-02	Research exposition (monographs, survey articles) pertaining to category theory

Keywords:

linear logic; complete lattices with sup-preserving associative binary relation; quantales; quantaloids; automata; tree automata; enriched category theory; *-autonomous categories