Hofmann, Karl H.; Morris, Sidney A.

The structure of compact groups. A primer for the student – A handbook for the expert. 3rd revised and augmented ed. (English) Zbl 1277.22001

de Gruyter Studies in Mathematics 25. Berlin: de Gruyter (ISBN 978-3-11-029655-6/hbk; 978-3-11-029679-2/ebook). xxii, 924 p. (2013).

The present book occupies the same position in the arena of compact groups as the three-volume work of *N. Dunford* and *J. T. Schwartz* [Linear operators. Part I: General theory. New York and London: Interscience Publishers (1958; Zbl 0084.10402); Linear operators. Part II: Spectral theory. Self adjoint operators in Hilbert space. New York and London: Interscience Publishers, a division of John Wiley & Sons (1963; Zbl 0128.34803); Linear operators. Part III: Spectral operators. New York etc.: Wiley- Interscience, a division of John Wiley & Sons (1971; Zbl 0243.47001)] does in the area of linear operators and the two-volume work of *C. W. Curtis* and *I. Reiner* [Methods of representation theory. With applications to finite groups and orders. Vol. I. New York etc.: John Wiley & Sons (1981; Zbl 0469.20001); ibid. Vol. II. New York etc.: John Wiley & Sons (1987; Zbl 0616.20001)] does in the realm of representation theory. It could be called the canon. Therefore, it is very pleasant to see its third edition to be published which has increased its size from 858 pages of the second edition (2006) (for a review, see Zbl 1139.22001) to 924 pages. This is due mainly to the addition of two new appendices. The first new appendix (Appendix 5) investigates the compact semigroup $P(G)$ of all probability measures on a compact group G under convolution. This third edition is more self-contained than the former editions, for the information in this appendix was crammed into a long-drawn exercise in Chapter 2 in the former two editions. The second new appendix (Appendix 6) is concerned with the representation of compact groups in terms of projective limits of certain well-ordered inverse systems, containing in particular a theorem stating that the underlying space of every compact group is supercompact.

One of the authors' mathematical philosophies is "the emphasis and application of Lie theory pervading the book from Chapter 5", by which they mean "a consistent use of not necessarily finite dimensional Lie algebras and the associated exponential function wherever it is feasible and advances the structural insight". Therefore, the authors mentionsome of their previous works [J. Lie Theory 21, No. 2, 347–383 (2011; Zbl 1226.22002); J. Group Theory 14, No. 6, 931–935 (2011; Zbl 1246.22003); The Lie theory of connected pro-Lie groups. A structure theory for pro-Lie algebras, pro-Lie groups, and connected locally compact groups. Zürich: European Mathematical Society (2007; Zbl 1153.22006)] as selected references in the preface.

Another significant refinement in this third edition is that it takes the paper [Topology Appl. 159, No. 9, 2235–2247 (2012; Zbl 1247.22008)] by *S. A. Antonyan* into account in Chapter 10, where actions of compact groups are concerned. As it is well-known, S. A. Antonyan found a serious gap in the proof by *J. Szenthe* [Acta Sci. Math. 36, 323–344 (1974; Zbl 0269.57019)] of the familiar result that a transitive action of a compact group G on a space X causes X to be a real analytic manifold, provided that X is locally contractible. The treatment of this point in the book is akin to the joint work of the first author with Kramer, which seems to remain unpublished.

Reviewer: Hirokazu Nishimura (Tsukuba)

MSC:

22-02	Research exposition (monographs, survey articles) pertaining to topological groups
22C05	Compact groups
22B05	General properties and structure of LCA groups
22E15	General properties and structure of real Lie groups
22E65	Infinite-dimensional Lie groups and their Lie algebras: general properties
54H11	Topological groups (topological aspects)

Keywords:

compact group; linear Lie group; compact Lie group; Lie algebra; profinite group; homotopy group; homology group; abelian topological group; covering space; free compact group

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