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Proof theory of paraconsistent quantum logic. (English) Zbl 06875349

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Minimal quantum logic or *orthologic* was introduced in [*G. Birkhoff* and *J. von Neumann*, Ann. Math. (2) 37, 823–843 (1936; JFM 62.1061.04); *G. Birkhoff* and *J. von Neumann*, Ann. Math. (2) 37, 823–843 (1936; Zbl 0015.14603)], and its Kripke-style semantics was provided in [*R. I. Goldblatt*, J. Philos. Log. 3, 19–35 (1974; Zbl 0278.02023)]. Its Gentzen-type sequent calculi have been studied in [*N. J. Cutland* and *P. F. Gibbins*, Log. Anal., Nouv. Sér. 25, 221–248 (1982; Zbl 0518.03029); *C. Faggian* and *G. Sambin*, Int. J. Theor. Phys. 37, No. 1, 31–37 (1998; Zbl 0904.03031); *H. Nishimura*, in: Handbook of quantum logic and quantum structures. Quantum logic. With a foreword by Anatolij Dvurečenskij. Amsterdam: Elsevier/North-Holland. 227–260 (2009; Zbl 1273.03089); *H. Nishimura*, Int. J. Theor. Phys. 33, No. 7, 1427–1443 (1994; Zbl 0809.03045); *H. Nishimura*, Int. J. Theor. Phys. 33, No. 1, 103–113 (1994; Zbl 0798.03062); *H. Nishimura*, J. Symb. Log. 45, 339–352 (1980; Zbl 0437.03034); *S. Tamura*, Kobe J. Math. 5, No. 1, 133–150 (1988; Zbl 0663.03050); *M. Takano*, Int. J. Theor. Phys. 34, No. 4, 649–654 (1995; Zbl 0824.03032)].

Belnap and Dunn's *paraconsistent four-valued logic* [*J. M. Dunn*, Philos. Stud. 29, No. 3, 149–168 (1976; Zbl 06943294); *N. D. Belnap jun.*, in: Mod. uses of multiple-valued logic, 5th int. Symp., Bloomington 1975, 5–37 (1977; Zbl 0417.03009); *N. D. Belnap jun.*, in: Mod. Uses of multiple-valued Logic, 5th int. Symp., Bloomington 1975, 5–37 (1977; Zbl 0424.03012)], a.k.a. Anderson and Belnap's *first-degree entailment* [*A. R. Anderson* and *N. D. Belnap jun.*, Entailment. The logic of relevance and necessity. Vol. I. Princeton, N. J.: Princeton University Press (1975; Zbl 0323.02030); *A. R. Anderson* et al., Entailment. The logic of relevance and necessity. Vol. II. Princeton, NJ: Princeton University Press (1992; Zbl 0921.03025)], is known to be equivalent to the $\{\wedge, \vee, \sim\}$-fragment of Nelson's *paraconsistent four-valued logic* [*D. Nelson*, J. Symb. Log. 14, 16–26 (1949; Zbl 0033.24304); *A. Almukdad* and *D. Nelson*, J. Symb. Log. 49, 231–233 (1984; Zbl 0575.03016)]. Its Gentzen-type sequent calculi have been investigated in [*J. M. Font*, Log. J. IGPL 5, No. 3, 413–440 (1997; Zbl 0871.03012); *J. M. Font*, Log. J. IGPL 7, No. 5, 671–672 (1999; Zbl 0937.03028); *N. Kamide* and *H. Wansing*, Proof theory of N4-paraconsistent logics. London: College Publications (2015; Zbl 06407640); *A. P. Pynko*, Math. Log. Q. 41, No. 4, 442–454 (1995; Zbl 0837.03019); Zbl 0323.02030].

Paraconsistent quantum logic, a hybrid of minimal quantum logic and paraconsistent four-valued logic, was introduced in [*M. L. Dalla Chiara* and *R. Giuntini*, Synthese 125, No. 1–2, 55–68 (2000; Zbl 0969.03070)]. A cut-free Gentzen-type sequent calculus for it was investigated in [Zbl 0904.03031] by extending the $\{\wedge, \vee\}$-fragment of basic logic in [*G. Sambin* et al., J. Symb. Log. 65, No. 3, 979–1013 (2000; Zbl 0969.03017)].

This paper introduces four cut-free Gentzen-type sequent calculi for paraconsistent quantum logic. The first is obtained from Takano's [Zbl 1273.03089; Zbl 0824.03032] by deleting some initial sequents and negated logical inference rules. The second comes from a Gentzen-type sequent calculus for the $\{\wedge, \vee\}$-fragment of basic logic by adding some negated logical inference rules. The third derives from a Gentzen-type sequent calculus for the $\{\wedge, \vee\}$-fragment of Aoyama's *weak sequent calculus* LB ["On a weak system of sequent calculus", J. Logical Philosophy 3, 29–37 (2003)] by adding some negated logical inference rules. The fourth, PQL, is obtained from the third by restricting the sequent definition, being obtainable from a Gentzen-type sequent calculus for *lattice logic* [*G. Restall* and *F. Paoli*, J. Symb. Log. 70, No. 4, 1108–1126 (2005; Zbl 1100.03049); *J. Schulte Moenting*, Algebra Univers. 12, 290–321 (1981; Zbl 0528.03029)] or from *finite sum-product logic* [*J. R. B. Cockett* and *R. A. G. Seely*, Theory Appl. Categ. 8, 63–99 (2001; Zbl 0969.03071)] by adding some negated logical inference rules. These four calculi are logically equivalent. A first-order predicate extension FPQL of PQL is shown to be decidable after the decidability of first-order *substructural logics* without contraction rules.

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