Bourke, John; **Lack, Stephen**

Braided skew monoidal categories. (English) Zbl 07161907

Theory Appl. Categ. 35, 19-63 (2020).

The notion of *skew monoidal category* was introduced in [*K. Szlachányi*, Adv. Math. 231, No. 3–4, 1694–1730 (2012; Zbl 1283.18006)] in order to deal with bialgebroids. It is, roughly speaking, obtained from the familiar notion of *monoidal category* by loosening isomorphisms on associativity and unit conditions. This paper answer the question whether there exists a sensible notion of braiding for skew monoidal categories, generalizing the classical theory of braided monoidal categories [*A. Joyal* and *R. Street*, Adv. Math. 102, No. 1, 20–78 (1993; Zbl 0817.18007)]. The authors give a notion of braiding on a skew monoidal category which is given by an invertible natural transformation

$$s:(AB)C\to(AC)B$$

pursuant to certain axioms. If B is a bialgebra, one gets a skew monoidal structure $\mathbf{Vect}[B]$ on the category $\mathbf{Vect}$ of vector spaces with product

$$X\bigstar Y=X\otimes B\otimes Y$$

and the ground field K as its unit. Bialgebroids give rise to, and can be characterized by certain skew monoidal categories [*K. Szlachányi*, Adv. Math. 231, No. 3–4, 1694–1730 (2012; Zbl 1283.18006)].

It is shown in Theorem 4.10 that braidings on the skew monoidal category are in bijection with *cobraidings* (a.k.a. *coquasitriangular structures* [*C. Kassel*, Quantum groups. New York, NY: Springer-Verlag (1995; Zbl 0808.17003); *R. Street*, Quantum groups. A path to current algebra. Cambridge: Cambridge University Press (2007; Zbl 1117.16031)]) on the bialgebra B. The theorem is presented as a specialized one of the main theorem Theorem 4.7 in §4 claiming that, given a monoidal comonad G on a monoidal category $\mathcal{C}$ abiding by a mild hypothesis, there is a bijection between braidings on the monoidal category $\mathcal{C}^G$ of coalgebras and braidings on the cowarped skew monoidal category $\mathcal{C}[G]$.

The other leading class of examples in the paper arises naturally if one attempts to study 2-categorical structures as strictly as possible, e.g., by considering the 2-category $\mathbf{FProd}_s$ of categories endowed with a choice of finite products and functors which *strictly* preserve them in place of $\mathbf{FProd}$ of categories with finite products and functors preserving finite products up to isomorphisms. The skew monoidal structure on $\mathbf{FProd}_s$ is much easier to construct [*J. Bourke*, J. Homotopy Relat. Struct. 12, No. 1, 31–81 (2017; Zbl 1417.18001)]. Indeed, they are exhibited in §6 after braided skew multicategories are introduced and it is shown how to pass from these to braided skew monoidal categories under the assumption of a representability condition in §5.

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MSC:

18M50 Bimonoidal, skew-monoidal, duoidal categories
18M15 Braided monoidal categories and ribbon categories
18N10 2-categories, bicategories, double categories
18N40 Homotopical algebra, Quillen model categories, derivators
16T10 Bialgebras

Keywords:

braiding; skew monoidal category; bialgebra; quasitriangular; 2-category

Full Text: Link

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