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**The 2-Chu-Dialectica construction and the polycategory of multivariable adjunctions.** (English) [\[Zbl 07163720\]](#)

Theory Appl. Categ. 35, 89-136 (2020).

On the one hand, the categorical Dialectica constructions were introduced by *V. C. V. de Paiva* [Contemp. Math. 92, 47–62 (1989; [Zbl 0675.03039](#))] as an abstraction of Gödel’s “Dialectica interpretation” [*K. Gödel*, Dialectica 12, 280–287 (1958; [Zbl 0090.01003](#))]. De Paiva’s categorical analysis revealed that it factors naturally through *J.-Y. Girard’s classical* linear logic [Theor. Comput. Sci. 50, 1–102 (1987; [Zbl 0625.03037](#))], which means a  $*$ -autonomous category from a categorical viewpoint. On the other hand, the Chu construction was introduced as a way to produce  $*$ -autonomous categories in [*M. Barr*,  $*$ -autonomous categories. With an appendix by Po-Hsiang Chu. Berlin-Heidelberg-New York: Springer-Verlag (1979; [Zbl 0415.18008](#))]. These two constructions have the same vein, and one formal functorial comparison was given in [*V. de Paiva*, Theory Appl. Categ. 17, 127–152 (2006; [Zbl 1123.18004](#))]. The principal objective in this paper is to give a new comparison by giving such a general construction that includes both Dialectica and Chu ones as special cases.

The difficulty in comparison lies in that their monoidal structures are quite distinct in spite of similarity between their underlying categories. To overcome this difficulty, the author uses multicategories in place of monoidal categories and polycategories [*M. E. Szabo*, Commun. Algebra 3, 663–689 (1975; [Zbl 0353.18008](#))] instead of  $*$ -autonomous categories. The polycategorical perspective allows one to exhibit both the Dialectica construction and the Chu construction as instances of one “2-Chu-Dialectica construction”, which besides includes the polycategory of polarized multivariable adjunctions at the other vertex.

Now is a brief synopsis of the paper in order. §2 defines the abstract input and also the output of the author’s 2-Chu-Dialectica construction, while §3 gives the 2-Chu-Dialectica construction. §4 shows how it specializes to one of the Dialectica constructions, while §5 shows how it specializes to the Chu construction. In §6 it specializes to the 2-Chu construction, and the result is enhanced to a poly double category of polarized multivariable adjunctions. §7 connects this construction to the multi double category of [*E. Cheng et al.*, J. K-Theory 13, No. 2, 337–396 (2014; [Zbl 1326.18005](#))].

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#### MSC:

18D10 Monoidal, symmetric monoidal and braided categories (MSC2010)

18D05 Double categories, 2-categories, bicategories and generalizations (MSC2010)

#### Keywords:

[multivariable adjunction](#); [polycategory](#); [Dialectica construction](#); [Chu construction](#)

**Full Text:** [Link](#)

#### References:

- [1] Tom Avery. Structure and Semantics. PhD thesis, University of Edinburgh, 2017. [arXiv:1708.01050](#).
- [2] Michael Barr.  $*$ -autonomous categories, volume 752 of Lecture Notes in Mathematics. Springer, 1979. · [Zbl 0874.18004](#)
- [3] Michael Barr.  $*$ -autonomous categories and linear logic. Mathematical Structures in Computer Science, 1(2):159178, 1991. · [Zbl 0777.18006](#)
- [4] Michael Barr. The Chu construction: history of an idea. Theory and Applications of Categories, 17(1):10-16, 2006. · [Zbl 1110.18001](#)
- [5] Bodil Biering. Dialectica interpretations: a categorical analysis. PhD thesis, IT University of Copenhagen, 2008. · [Zbl 1155.03047](#)
- [6] Renato Betti and A. John Power. On local adjointness of distributive bicategories. Boll. Un. Mat. Ital. B (7), 2(4):931-

- 947, 1988. · [Zbl 0665.18007](#)
- [7] Eugenia Cheng, Nick Gurski, and Emily Riehl. Cyclic multicategories, multivariable adjunctions and mates. *Journal of K-Theory*, 13(2):337-396, 2014. · [Zbl 1326.18005](#)
- [8] Po-Hsiang Chu. Constructing  $\ast$ -autonomous categories. M. Sc. thesis, McGill University, 1978.
- [9] Po-Hsiang Chu. Constructing  $\ast$ -autonomous categories. In  $\ast$ -autonomous categories, volume 752 of *Lecture Notes in Mathematics*, chapter Appendix.
- [10] J.R.B. Cockett, J. Koslowski, and R.A.G. Seely. Introduction to linear bicategories. *Mathematical Structures in Computer Science*, 2:165-203, 2000. · [Zbl 0991.18007](#)
- [11] J.R.B. Cockett, J. Koslowski, and R.A.G. Seely. Morphisms and modules for poly-bicategories. *Theory and Applications of Categories*, 11(2):15-74, 2003. · [Zbl 1022.18006](#)
- [12] J.R.B. Cockett and R.A.G. Seely. Proof theory for full intuitionistic linear logic, bilinear logic, and mix categories. *Theory and Applications of Categories*, · [Zbl 0879.03022](#)
- [13] Robin Cockett and Robert Seely. Weakly distributive categories. *Journal of Pure and Applied Algebra*, 114(2):133-173, 1997. · [Zbl 0867.18008](#)
- [14] J.R.B. Cockett and R.A.G. Seely. Polarized category theory, modules, and game semantics. *Theory and Applications of Categories*, 18(2):4-101, 2007. · [Zbl 1117.03071](#)
- [15] G.S.H. Cruttwell and Michael Shulman. A unified framework for generalized multicategories. *Theory Appl. Categ.*, 24:580-655, 2010. arXiv:0907.2460. · [Zbl 1220.18003](#)
- [16] Brian Day. On closed categories of functors. In *Reports of the Midwest Category Seminar, IV*, *Lecture Notes in Mathematics*, Vol. 137, pages 1-38. Springer,
- [17] Gabriel C. Drummond-Cole and Philip Hackney. Dwyer-Kan homotopy theory for cyclic operads. arXiv:1809.06322v1, 2018.
- [18] Valeria de Paiva. The Dialectica categories. In J. Gray and A. Scedrov, editors, *Proc. of Categories in Computer Science and Logic*, Boulder, CO, 1987. · [Zbl 06856650](#)
- [19] Valeria C. V. de Paiva. A Dialectica-like model of linear logic. In David H. Pitt, David E. Rydeheard, Peter Dybjer, Andrew M. Pitts, and Axel Poign´e, editors, · [Zbl 1407.03076](#)
- [20] Valeria de Paiva. Categorical multirelations, linear logic and Petri nets. Technical Report 225, University of Cambridge, 1991.
- [21] Valeria de Paiva. Dialectica and Chu constructions: cousins? *Theory and Applications of Categories*, 17(7):127-152, 2006. · [Zbl 1123.18004](#)
- [22] Brian Day and Ross Street. Monoidal bicategories and Hopf algebroids. *Adv. Math.*, 129(1):99-157, 1997. · [Zbl 0910.18004](#)
- [23] Brian Day and Ross Street. Quantum categories, star autonomy, and quantum groupoids. In *Galois Theory, Hopf Algebras, and Semiabelian Categories*, · [Zbl 1067.18006](#)
- [24] J.M. Egger. The Frobenius relations meet linear distributivity. *Theory and Applications of Categories*, 24(2):25-38, 2010. · [Zbl 1226.03066](#)
- [25] Samuel Eilenberg and G. Max Kelly. Closed categories. In *Proc. Conf. Categorical Algebra (La Jolla, Calif., 1965)*, pages 421-562. Springer, New York, 1966.
- [26] Richard Garner. n-Category Café comment: “Re: Monoidal closed categories and their deviant relatives”. [https://golem.ph.utexas.edu/category/02/monoidal\\_closed\\_categories\\_and.html#c022449](https://golem.ph.utexas.edu/category/02/monoidal_closed_categories_and.html#c022449), 2009.
- [27] Jean-Yves Girard. Linear logic. *Theoretical Computer Science*, 50(1):1- 101, 1987. · [Zbl 0625.03037](#)
- [28] E. Getzler and M. M. Kapranov. Cyclic operads and cyclic homology. In *Geometry, topology, & physics*, *Conf. Proc. Lecture Notes Geom. Topology*, IV, · [Zbl 0883.18013](#)
- [29] Marco Grandis and Robert Pare. Adjoints for double categories. *Cah. Topol. G´eom. Diff´er. Cat´eg.*, 45(3):193-240, 2004. · [Zbl 1063.18002](#)
- [30] John W. Gray. Closed categories, lax limits and homotopy limits. *J. Pure Appl. Algebra*, 19:127-158, 1980. · [Zbl 0462.55008](#)
- [31] P. Gabriel and F. Ulmer. *Lokal pr´esentierbare Kategorien*, volume 221 of *Lecture Notes in Mathematics*. Springer, 1971.
- [32] Ren´e Guitart. Trijunctions and triadic Galois connections. *Cah. Topol. G´eom. Diff´er. Cat´eg.*, 54(1):13-27, 2013.
- [33] Pieter Hofstra. The dialectica monad and its cousins. In *Models, Logics, and Higher-dimensional Categories: A Tribute to the Work of Mih´aly Makkai*, pages 107-137. American Mathematical Society, 09 2011. · [Zbl 1243.03074](#)
- [34] Mark Hovey. *Model Categories*, volume 63 of *Mathematical Surveys and Monographs*. American Mathematical Society, 1999.
- [35] Philip Hackney, Marcy Robertson, and Donald Yau. Higher cyclic operads. *Algebraic & Geometric Topology*, 19:863-940, 2019. · [Zbl 07075116](#)
- [36] J.M.E. Hyland. Proof theory in the abstract. *Annals of Pure and Applied Logic*, 114:43-78, 2002. · [Zbl 1007.03056](#)
- [37] G. M. Kelly and Ross Street. Review of the elements of 2-categories. In *Category Seminar (Proc. Sem., Sydney, 1972/1973)*, volume 420 of *Lecture Notes in Math.*, pages 75-103. Springer, Berlin, 1974.
- [38] F. William Lawvere. Equality in hyperdoctrines and comprehension schema as an adjoint functor. In *Applications of*

- [39] F. William Lawvere. Adjointness in foundations. *Repr. Theory Appl. Categ.*, 16:1-16 (electronic), 2006. Reprinted from *Dialectica* 23(1969). · [Zbl 1114.18002](#)
- [40] Duško Pavlović. Chu I: cofree equivalences, dualities, and  $*$ -autonomous categories. *Math. Struct. in Comp. Science*, 11, 1993.
- [41] Vaughan Pratt, John Baez, and Michael Barr. Does duality categorify? Discussion on the categories mailing list: <http://www.mta.ca/~cat-dist/archive/2006/06-4andhttp://www.mta.ca/~cat-dist/archive/2006/06-5>, 2006.
- [42] Uday S. Reddy. Acceptors as values. <http://www.cs.bham.ac.uk/~udr/>, 1991.
- [43] Emily Riehl. Monoidal algebraic model structures. *Journal of Pure and Applied Algebra*, 217(6):1069-1104, 2013. · [Zbl 1275.55014](#)
- [44] Michael Shulman. Enriched indexed categories. *Theory and Applications of Categories*, 28(21):616-695, 2013. · [Zbl 1273.18023](#)
- [45] Michael Shulman. Contravariance through enrichment. *Theory and Applications of Categories*, 33(5):95-130, 2018. [arXiv:1606.05058](https://arxiv.org/abs/1606.05058). · [Zbl 1393.18002](#)
- [46] Michael Shulman.  $*$ -autonomous categories are Frobenius pseudomonoids. In preparation, 2019.
- [47] F. Guilln Santos, V. Navarro, P. Pascual, and A. Roig. Moduli spaces and formal operads. *Duke Math. J.*, 129(2):291-335, 08 2005. · [Zbl 1120.14018](#)
- [48] Ross Street. Fibrations and Yoneda's lemma in a 2-category. In *Category Seminar (Proc. Sem., Sydney, 1972/1973)*, pages 104-133. *Lecture Notes in*
- [49] Ross Street. Frobenius monads and pseudomonoids. *J. Math. Phys.*, 45(10):3930-3948, 2004. · [Zbl 1071.18006](#)
- [50] Ross Street and Robert Walters. Yoneda structures on 2-categories. *J. Algebra*, 50(2):350-379, 1978. · [Zbl 0401.18004](#)
- [51] M.E. Szabo. Polycategories. *Communications in Algebra*, 3(8):663-689, 1975.
- [52] Mark Weber. Yoneda structures from 2-toposes. *Appl. Categ. Structures*, 15(3):259-323, 2007. · [Zbl 1125.18001](#)
- [53] R. J. Wood. Abstract proarrows. I. *Cahiers Topologie G' eom. Diff' erentielle*,

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