Grandis, Marco
Higher dimensional categories. From double to multiple categories. (English) Zbl 07082237
Hackensack, NJ: World Scientific (ISBN 978-981-12-0510-1/hbk; 978-981-12-0512-5/ebook). xi, 522 p.
(2020).

Higher dimensional categories began with (strict) double and multiple categories, introduced and investigated in [*C. Ehresmann*, Rev. Unión Mat. Argent. 20, 194–209 (1962; Zbl 0117.26104); Ann. Sci. Éc. Norm. Supér. (3) 80, 349–426 (1963; Zbl 0128.02002); C. R. Acad. Sci., Paris 256, 1891–1894 (1963; Zbl 0132.25703); Catégories et structures. Paris: Dunod. (1965; Zbl 0192.09803); *A. Bastiani* and *C. Ehresmann*, Cah. Topologie Géom. Différ. Catégoriques 15, 215–292 (1974; Zbl 0332.18005); *A. Ehresmann* and *C. Ehresmann*, Cah. Topologie Géom. Différ. Catégoriques 19, 295–333 (1978; Zbl 0415.18005)]. This book investigates double categories, n-tuple categoreis and multiple categories with their weak and lax versions. The main part of the book is divided into two parts.

The first part (Chapters 1–5) is concerned with double categories. Chapter 1 is a review of classical category theory. Chapter 2 studies 2-dimensional categorical structures, from ordered categories to 2-categories and bicategories. Chapters from 3 through 5 are concerned with weak double categories in general. Chapter 3 deals with strict and weak double categories, together with their functors and transformations. Chapters 4 and 5 address adjunctions and limits respectively for weak double categories after [the author, Ann. Mat. Pura Appl. (4) 157, 199–244 (1990; Zbl 0739.18004); Theory Appl. Categ. 18, 321–347 (2007; Zbl 1127.18003)].

The second part (Chapters 6–8) is engaged in the study of multiple categories, which are an extension of double categories to higher dimensions with one strict composition in direction 0 and various (possibly weak) compositions in positive directions. Beginning with a triple category built on the double category $\mathbb{D}$bl of weak double categories, Chapter 6 defines strict, weak and lax multiple categories. The author's main form, partially lax, is a *chiral multiple category*. *Intercategories*, a laxer form of multiple categories, are also considered on the lines of [*M. Grandis* and *R. Paré*, Theory Appl. Categ. 30, 1215–1255 (2015; Zbl 1387.18012); J. Pure Appl. Algebra 221, No. 5, 999–1054 (2017; Zbl 1375.18036)]. Chapter 7, concerned with adjunctions in chiral multiple categories, is based upon [*M. Grandis* and *R. Paré*, Categ. Gen. Algebr. Struct. Appl. 6, No. 1, 85–105 (2017; Zbl 1386.18019)]. Chapter 8 deals with lax and colax monads on chiral multiple categories and their relationship with multiple adjunctions. The main non-standard results are concerned with the idempotent case. Multiple limits are considered only in particular cases, the reader being referred to [*M. Grandis* and *R. Pare*, Cah. Topol. Géom. Différ. Catég. 58, No. 1, 3–48 (2017; Zbl 1387.18013)] for a general study.

The book is accompanied by three appendices. The first appendix (Appendix A) addresses applications of weak double categories in homological algebra [the author, Homological algebra. The interplay of homology with distributive lattices and orthodox semigroups. Hackensack, NJ: World Scientific (2012; Zbl 1280.18001)] and algebraic topology [the author, Zbl 1127.18003; J. Homotopy Relat. Struct. 3, No. 1, 273–308 (2008; Zbl 1185.18005); Theory Appl. Categ. 18, 602–630 (2007; Zbl 1136.18004)]. The second appendix (Appendix B) is concerned with symmetries in cubical sets and weak cubical categories on the lines of [the author, Zbl 1127.18003; Zbl 1136.18004; Cah. Topol. Géom. Différ. Catég. 50, No. 2, 102–143 (2009; Zbl 1188.18003)]. The third appendix (Appendix C) contains solutions or hints for the exercises of the text.

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MSC:

18-01 Introductory exposition (textbooks, tutorial papers, etc.) from category theory
18Dxx Categorical structures

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