Fujii, Soichiro

A unified framework for notions of algebraic theory. (English) Zbl 07136505

Theory Appl. Categ. 34, 1246-1316 (2019).

This paper, consisting of ten sections, aims to provide a formal core throughout a diversity of various notions of algebraic theory. §3 reviews several known notions of algebraic theory, say, clones [*W. Taylor*, NATO ASI Ser., Ser. C, Math. Phys. Sci. 389, 507–530 (1993; Zbl 0792.08005)], non-symmetric operads [*J. P. May*, The geometry of iterated loop spaces. Berlin-Heidelberg-New York: Springer-Verlag (1972; Zbl 0244.55009)], symmetric operads, PROPs [*S. MacLane*, Bull. Am. Math. Soc. 71, 40–106 (1965; Zbl 0161.01601)], monads [*S. Eilenberg* and *J. C. Moore*, Ill. J. Math. 9, 381–398 (1965; Zbl 0135.02103); *F. E. J. Linton*, in: Proc. Conf. Categor. Algebra, La Jolla 1965, 84–94 (1966; Zbl 0201.35003)] and generalized operads [*A. Burroni*, Cah. Topologie Géom. Différ. Catégoriques 12, 215–321 (1971; Zbl 0246.18007); *G. M. Kelly*, Lond. Math. Soc. Lect. Note Ser. 177, 163–190 (1992; Zbl 0789.18007); *C. Hermida*, Adv. Math. 151, No. 2, 164–225 (2000; Zbl 0960.18004); *T. Leinster*, Higher operads, higher categories. Cambridge: Cambridge University Press (2004; Zbl 1160.18001)]. §4 is concerned with syntactical aspects of notions of algebraic theory, identifying a notion of algebraic theory, called a *metatheory*, with a monoidal category and an algebraic theory of the notion of algebraic theory, called a *theory*, with a monoid in the monoidal category. §5–§7 are concerned with semantical aspects of metatheories, a notion of model, being defined with respect to the metatheory $\mathcal{M}$ in a category $\mathcal{C}$ in due consideration of enrichments (§5) and oplax actions (§6) and called a *metamodel*. §8 addresses morphisms between metatheories in order to compare different notions of algebraic theory. §9 constructs a structure-semantics adjunction for an arbitrary metatheory and an arbitrary metamodel of it. §10 gives an abstract characterization of the categories of models after the similar result for Eilenberg-Moore category of a monad [*R. Street*, J. Pure Appl. Algebra 2, 149–168 (1972; Zbl 0241.18003)]. It is established that categories of models are to be characterized by a certain universal property in terms of *pseudo double categories* [*M. Grandis* and *R. Paré*, Cah. Topologie Géom. Différ. Catégoriques 40, No. 3, 162–220 (1999; Zbl 0939.18007)].

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MSC:

18C10 Theories (e.g., algebraic theories), structure, and semantics
18D05 Double categories, 2-categories, bicategories and generalizations (MSC2010)

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