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An abstract elementary class nonaxiomatizable in $L_{\infty,\kappa}$. (English) Zbl 07106194

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It is difficult to determine in general which categories are to be obtained as categories of points of a topos. The question is of paramount interest to model theorists because of the following familiar theorem.

Theorem. A category is the category of points of a topos iff it is the category of models of geometric theory and morphisms of structures between them.

The theorem claims that every geometric theory admits a clasifying topos and that every topos is the classifying topos of such a theory. Given a theory making use of axioms in $L_{\infty,\omega}$, the process of *Morleyisation* allows it to turn into a geometric theory at the price of changing the signature [*P. T. Johnstone*, Sketches of an elephant. A topos theory compendium. I. Oxford: Clarendon Press (2002; Zbl 1071.18001); *P. T. Johnstone*, Sketches of an elephant. A topos theory compendium. II. Oxford: Clarendon Press (2002; Zbl 1071.18002; *C. Espíndolaar*, "Infinitary generalizations of Deligne's completeness theorem", Preprint, arXiv:1709.01967]. This article settles the following problem posed by *T. Beke* and *J. Rosický* [Ann. Pure Appl. Logic 163, No. 12, 2008–2017 (2012; Zbl 1315.03049)]:

Problem. Show that the category of uncountable sets and monomorphisms between can not be obtained as the category of points of a topos. Or give an example of an abstract elementary class that does not arise as the category of points of a topos.

The main result of the article (Theorem 3.3) goes as follows:

Theorem. Let κ be a regular cardinal. For any cardinal $\lambda > \kappa$, the category $Set^m_{\geqslant\lambda}$ of sets of cardinality at least λ and monomorphisms between them is not equivalent to the category of κ-points of a κ-topos.

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MSC:

18C35 Accessible and locally presentable categories
18C10 Theories (e.g., algebraic theories), structure, and semantics
03G30 Categorical logic, topoi
03C48 Abstract elementary classes and related topics

Keywords:

toposes; points of toposes; Scott topology; abstract elementary classes

Full Text: DOI

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