Kong, Liang; **Tian, Yin**; **Zhou, Shan**

The center of monoidal 2-categories in 3+1D Dijkgraaf-Witten theory. (English) Zbl 07146130

Adv. Math. 360, Article ID 106928, 25 p. (2020).

It was long ago [*J. C. Baez* and *M. Neuchl*, Adv. Math. 121, No. 2, 196–244 (1996; Zbl 0855.18008); *M. Kapranov* and *V. Voevodsky*, J. Pure Appl. Algebra 92, No. 3, 241–267 (1994; Zbl 0791.18010)] that the notion of the center of a monoidal 2-category was introduced. The principal objective in this paper is to compute the centers of non-trivial monoidal 2-categories in the simplest case.

Let $\mathcal{V}$ be the 1-category of finite-dimensional vector spaces over $\mathbb{C}$. Let G be a finite group and $\omega \in Z^4(G, \mathbb{C}^\times)$ a 4-cocycle. Let $2\mathrm{Vec}_G^\omega$ be the 2-category of G-graded 1-categories of finite semisimple $\mathcal{V}$-module categories endowed with a ω-twisted monoidal structure, making $2\mathrm{Vec}_G^\omega$ a non-strict monoidal 2-category. The goal is to compute the center of $2\mathrm{Vec}_G^\omega$ as a braided monoidal 2-category, which can be seen as 3+1D Dijkgraaf-Witten theory for a finite group G [*R. Dijkgraaf* and *E. Witten*, Commun. Math. Phys. 129, No. 2, 393–429 (1990; Zbl 0703.58011)].

The paper contains three principal results. The first principal result (Theorem 2.2) is that the center of a monoidal 2-category is a braided monoidal 2-category. The second principal result (Theorem 3.3) is an explicit computation of the center $\mathcal{Z}(2\mathrm{Vec}_G^\omega)$ as a 2-categorical generalization of *S. Willerton*'s [Algebr. Geom. Topol. 8, No. 3, 1419–1457 (2008; Zbl 1154.57029)] explicit computation of $\mathcal{Z}(1\mathrm{Vec}_G^\chi)$ with $\chi \in Z^3(G, \mathbb{C}^\times)$. The third principal result (Theorem 3.13) is that the sylleptic center of $\mathcal{Z}(2\mathrm{Vec}_G^\omega)$ is trivial.

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MSC:

18D05 Double categories, 2-categories, bicategories and generalizations (MSC2010)
81T45 Topological field theories in quantum mechanics

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braided monoidal 2-categories; Drinfeld center; topological quantum field theory

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