Dabrowski, Yoann; **Kerjean, Marie**
Models of linear logic based on the Schwartz ε-product. (English) Zbl 07146447
Theory Appl. Categ. 34, 1440-1525 (2019).

Linear logic (LL) was introduced by *J.-Y. Girard* in [Theor. Comput. Sci. 50, 1–102 (1987; Zbl 0625.03037)]. It was in [*T. Ehrhard* and *L. Regnier*, Theor. Comput. Sci. 309, No. 1–3, 1–41 (2003; Zbl 1070.68020); ibid. 364, No. 2, 166–195 (2006; Zbl 1113.03054)] that introduced was *Differential linear logic* (DiLL), natural models of which are required to find non-linear proofs interpreted by some classes of smooth maps, paving the way towards new computational interpretations of functional analytic constructions and a denotational interpretation of continuous or infinite data objects. A consequent categorical analysis was carried out by Blute, Cockett and Seely in [*R. F. Blute* et al., Theory Appl. Categ. 22, 622–672 (2009; Zbl 1262.18004); Math. Struct. Comput. Sci. 16, No. 6, 1049–1083 (2006; Zbl 1115.03092)], giving models of *intuitionistic* DiLL. This paper looks for models of *classical* DiLL, in which spaces should equal some double dual.

The paper is divided into two parts. The first part (§2-§5) focuses on building several *-autonomous categories. Identifying a proper notion of duality is indispensable for an interesting analytic tensor product. From an analytical viewpoint, the inductive tensor product is too weak to deal with extensions to completions, so that the weak dual or the Mackey dual, dealt with in [*M. Kerjean*, Log. Methods Comput. Sci. 12, No. 1, Paper No. 3, 23 p. (2016; Zbl 06554179)] is not strong enough. The authors find that the Arens dual and its strongly related tensor product called the ε-product exploited in [*L. Schwartz*, Ann. Inst. Fourier 7, 1–141 (1957; Zbl 0089.09601); Ann. Inst. Fourier 8, 1–209 (1958; Zbl 0089.09801)] are appropriate, being intimately related with nuclearity and Grothendieck's approximation property. It is shown that most of their general properties are nicely deducible from a very general *-autonomous category explained at the end of §3. The first model the authors get is seriously lacking in self-duality of the notion of locally convex space and notices that adjoining a bornology with weak compatibility conditions enables one to get a framework in which building a *-autonomous category is almost tautological. In order to obtain a *-autonomous category of locally convex spaces, one is required to impose some completeness condition for associativity maps of the ε-product and then to make the Arens dual compatible with some completion process with another duality functor of duals isomorphic to triple duals, which is exploited in §3 and §4 to obtain two extreme cases. §5 aims complementarily to find a *-autonomous framework that is well-suited for convenient smoothness, having to combine Mackey completeness with a Schwartz space property. The authors investigate in more detail the two extreme cases again, correponding to familiar functional analytic conditions, both invented by Grothendieck, namely Schwartz topologies and the subclass of nuclear topologies. At the end of the first part, one has a kind of generic methodology in producing *-autonomous categories of locally convex spaces from a kind of universal one dealt with in §2.

The second part (§6 and §7) develops a theory of variants of conveniently smooth maps being restricted to allow for continuous and not only bounded differentials. Starting with the conventional smoothness setting,§6 gives two denotational models of LL on the same *-autonomous category, with the same cartesian closed category of conveniently smooth maps, but with two distinct comonads. The authors use dialogue categories, but not via the models of tensor logic in [*P.-A. Melliès* and *N. Tabareau*, Ann. Pure Appl. Logic 161, No. 5, 632–653 (2010; Zbl 1223.03048)], but rather with a variant in order to retain cartesian closedness of the category equipped with non-linear maps as morphisms. §7 extend the models in §6 to models of full DiLL. After merging dialogue categories with differential λ-categories of [*A. Bucciarelli* et al., Electron. Notes Theor. Comput. Sci. 265, 213–230 (2010; Zbl 1342.68048)], the authors give three different models of DiLL, one on k-reflexive spaces (§7.13) and two being on the same of ρ-reflexive spaces with -smooth maps (§7.7), by remarking that the properties of ρ-smooth maps in order to get a DiLL model are reducible to properties of conveniently smooth maps under the extra continuity conditions on derivatives in a general categorical way.

The clarification of a natural way to get *-autonomous categories within an analytic setting suggests that one should reconsider familiar models in [*J.-Y. Girard*, Theor. Comput. Sci. 227, No. 1–2, 275–297 (1999; Zbl 0952.03025); Electronic Notes in Theoretical Computer Science 3, 7 p. (1996; Zbl 0909.68115)] from a more analytic viewpoint, leading the way to exploit the flourishing operator space theory in logic on the lines of [*J.-Y. Girard*, Lond. Math. Soc. Lect. Note Ser. 316, 346–381 (2004; Zbl 1073.03036)]. The strong interplay between logic, physics and functional analysis is indispensable with due regard to the more and more extensive use of conventional analysis in the study of quantum gravity from a standpoint of algebraic quantum field theory [*R. Brunetti* et al., Commun. Math. Phys. 345, No. 3, 741–779 (2016; Zbl 1346.83001)]. DiLL wemt only part way in considering smooth maps on linear spaces, rather than smooth maps on some type of smooth manifold. By providing well-behaved ?-monads, the authors suggest to try using ?-algebras for k-reflexive or ρ-reflexive spaces as a starting point to capture better infinite dimensional features than the usual Cahier topos. From a mathematical standpoint, this means merging recent advances in derived geometry with infinite dimensional analysis. From a physical standpoint, this suggests comparing recent homotopical approaches [*M. Benini* et al., Commun. Math. Phys. 359, No. 2, 765–820 (2018; Zbl 1423.14007)] with applications of the Batalin-Vilkovisky formalism [*K. Fredenhagen* and *K. Rejzner*, Commun. Math. Phys. 317, No. 3, 697–725 (2013; Zbl 1263.81245); Commun. Math. Phys. 314, No. 1, 93–127 (2012; Zbl 1418.70034)]. From a logical standpoint, this entails comprehension of the interplay between intuitionistic dependent type theory and linear logic. Since linear logic nicely captures infinite dimensional features, this finally proposes a strong interplay between parametrized analysis in homotopy theory and parametrized versions of linear logic [*P.-L. Curien* et al., in: Proceedings of the 43rd annual ACM SIGPLAN-SIGACT symposium on principles of programming languages, POPL '16, St. Petersburg, FL, USA, January 20–22, 2016. New York, NY: Association for Computing Machinery (ACM). 44–56 (2016; Zbl 1347.68078)].

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MSC:

03B47	Substructural logics (including relevance, entailment, linear logic, Lambek calculus, BCK and BCI logics)
18C50	Categorical semantics of formal languages
18D15	Closed categories (closed monoidal and Cartesian closed categories, etc.)
46A20	Duality theory for topological vector spaces
46M05	Tensor products in functional analysis
46E50	Spaces of differentiable or holomorphic functions on infinite-dimensional spaces
68Q55	Semantics in the theory of computing

Keywords:

topological vector spaces; $*$-autonomous and dialogue categories; differential linear logic

Full Text: Link

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