Bergner, Julia E.; **Osorno, Angélica M.**; **Ozornova, Viktoriya**; **Rovelli, Martina**; **Scheimbauer, Claudia I.**

The edgewise subdivision criterion for 2-Segal objects. (English) Zbl 07144487

Proc. Am. Math. Soc. 148, No. 1, 71-82 (2020).

The edgewise subdivision of a simplical space is a construction leaving the geometric realization unchanged but having the effect of decomposition of the simplicial space into more simplices. It appeared first in [*G. Segal*, Invent. Math. 21, 213–221 (1973; Zbl 0267.55020)], while *F. Waldhausen* [Lect. Notes Math. 1126, 318–419 (1985; Zbl 0579.18006)] used this construction to establish the equivalence of the $S_\bullet$-construction and the Q-construction in algebraic K-theory. This paper applies this construction to the 2-Segal spaces of [*T. Dyckerhoff* and *M. Kapranov*, Higher Segal spaces (to appear). Cham: Springer (2019; Zbl 07103772)], closely related with the decomposition spaces of [*I. Gálvez-Carrillo* et al., Adv. Math. 331, 952–1015 (2018; Zbl 1403.00023); ibid. 333, 1242–1292 (2018; Zbl 1403.18016); ibid. 334, 544–584 (2018; Zbl 1403.18017)]. In [*J. E. Bergner* et al., Topology Appl. 235, 445–484 (2018; Zbl 1422.55036); "2-Segal objects and the Waldhausen construction", `arXiv:1809.10924`] the authors demonstrated that any 2-Segal space abdiding by a unitality condition comes from such a construction for a suitably general input. This paper lies in the more general context of 2-Segal objects in any combinatorial model category. The main result (Theorem 2.11) goes as follows:

Theorem. Let X be a simplicial object in a combinatorial model category $\mathcal{M}$. Then X is a 2-Segal object iff its edgewise subdivision $\mathrm{esd}(X)$ is a Segal object.

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MSC:

18D35 Structured objects in a category (MSC2010)
18G30 Simplicial sets; simplicial objects in a category (MSC2010)
19D10 Algebraic K-theory of spaces
55U10 Simplicial sets and complexes in algebraic topology

Full Text: DOI

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