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Infinitary first-order categorical logic. (English) Zbl 06986590

Ann. Pure Appl. Logic 170, No. 2, 137-162 (2019).

C. R. Karp [Languages with expressions of infinite length. Amsterdam: North-Holland Publishing Company (1964; Zbl 0127.00901)] obtained completeness theorems for classical infinitary propositional and first-order logics with Hilbert-type systems, while a related development with Gentzen-type systems were to be seen in [*S. Maehara* and *G. Takeuti*, J. Math. Soc. Japan 13, 357–370 (1962; Zbl 0108.00203)]. *M. E. Nadel* [Ann. Math. Logic 14, 159–191 (1978; Zbl 0406.03055)] developed infinitary intuitionistic propositional logic for countable many conjunctions/disjunctions to establish its completeness with respect to the infinitary version of Kripke semantics. *M. Makkai* [Ann. Pure Appl. Logic 47, No. 3, 225–268 (1990; Zbl 0711.03030)] considered infinitary regular theories together with a corresponding completeness result.

The principal objective in this paper is to establish completeness for infinitary intuitionistic propositional and first-order logics with respect to infinitary Kripke semantics, as well as for sheaf and categorical models. The desired completeness is obtained by manipulating sheaf models in place of Henkin's method of adding constants [*L. Henkin*, J. Symb. Log. 14, 159–166 (1949; Zbl 0034.00602)]. The crucial contribution is the identification of the transfinite transitivity property, being a generalization of the transitivity property for Grothendieck topologies, as the appropriate categorical property leading to the axiomatic treatment proposed.

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MSC:

03G30 Categorical logic, topoi
03C75 Other infinitary logic
18B25 Topoi
18C50 Categorical semantics of formal languages

Keywords:

categorical logic; infinitary logics; completeness theorems; sheaf models; large cardinals

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