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Integral categories and calculus categories. (English) [Zbl 1408.18012] *Math. Struct. Comput. Sci.* 29, No. 2, 243–308 (2019).

The two fundamental theorems of classical calculus reveal the relation between differentiation and integration:

$$\frac{d(\int_a^t f(u)du)}{dt}(x) = f(x)$$

$$\int_a^b \frac{df(t)}{dt}(x)dx = f(b) - f(a)$$

T. Ehrhard and *L. Regnier* [*Theor. Comput. Sci.* 309, No. 1–3, 1–41 (2003; Zbl 1070.68020); ibid. 364, No. 2, 166–195 (2006; Zbl 1113.03054)] introduced the differential λ -calculus and differential proof nets. In [*Math. Struct. Comput. Sci.* 16, No. 6, 1049–1083 (2006; Zbl 1115.03092)], *R. F. Blute* et al. introduced differential categories for modelling Ehrhard and Regnier’s differential linear logic, which has been followed by [*R. Blute* et al., *Cah. Topol. Géom. Différ. Catég.* 52, No. 4, 253–268 (2011; Zbl 1254.13026); *Theory Appl. Categ.* 30, 620–686 (2015; Zbl 1330.18009); *Cah. Topol. Géom. Différ. Catég.* 57, No. 4, 243–279 (2016; Zbl 1364.13026); *Theory Appl. Categ.* 22, 622–672 (2009; Zbl 1262.18004); *J. R. B. Cockett* et al., *Theory Appl. Categ.* 25, 537–613 (2011; Zbl 1260.14004); *N. Yu. Burban* and *O. L. Horbachuk*, *Mat. Stud.* 35, No. 2, 121–127 (2011; Zbl 1304.18003); *M. P. Fiore*, *Lect. Notes Comput. Sci.* 4583, 163–177 (2007; Zbl 1215.03072); *J. Laird* et al., *Inf. Comput.* 222, 247–264 (2013; Zbl 1269.03062); *R. Blute* et al., *Cah. Topol. Géom. Différ. Catég.* 53, No. 3, 211–232 (2012; Zbl 1281.46061)].

T. Ehrhard [*Math. Struct. Comput. Sci.* 28, No. 7, 995–1060 (2018; Zbl 06914140)] observed that in certain $*$ -autonomous categories of an appropriate structure making it a differential category, it is possible to construct an integral transformation with an inverse behavior to the deriving transformation provided that a certain natural transformation (called J) is a natural isomorphism, and showed that every differential function satisfies the second fundamental theorem of calculus under the condition that the deriving transformation adheres to the Taylor property.

Following Ehrhard’s considerations and taking the older notion of a Rota-Baxter algebra [*G. Baxter*, *Pac. J. Math.* 10, 731–742 (1960; Zbl 0095.12705); *L. Guo*, *An introduction to Rota-Baxter algebra*. Somerville, MA: International Press; Beijing: Higher Education (2012; Zbl 1271.16001); *G. C. Rota*, *Bull. Am. Math. Soc.* 75, 325–329 (1969; Zbl 0192.33801); *Bull. Am. Math. Soc.* 75, 330–334 (1969; Zbl 0319.05008)] into account, the authors purpose to axiomatize integration irrespective of differentiation, resulting in integral categories (§3). By combining integral categories and differential categories (surveyed in §4) compatibly, we get the notion of calculus category (§5). §6 is devoted to exploring what it means for a differential category to have antiderivatives. §7 gives examples.

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MSC:

- 18D10 Monoidal, symmetric monoidal and braided categories
- 18F99 Categories and geometry
- 03F52 Linear logic and other substructural logics
- 03B40 Combinatory logic; lambda-calculus

Cited in 2 Documents

Keywords:

differentiation; integration; fundamental theorems of classical calculus; differential lambda calculus; differential proof nets; integral category; differential category; calculus category; Rota-Baxter algebra

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