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Products and coequalizers in pointed categories. (English) Zbl 07141212
Theory Appl. Categ. 34, 1386-1400 (2019).

This paper investigates the property (P) that binary products commute with arbitrary coequalizers in pointed categories, being concerned with pointed varieties, i.e., varieties possessing a unique constant. It is shown (Theorem 2.16) that a pointed variety satisfies the condition (P) iff there exist integers $m \geqslant 0$ and $n \geqslant 1$ such that its theory admits binary terms $b_i(x,y)$ and unary terms $c_i(x)$ for each $1 \leqslant i \leqslant m$ and $(m+2)$-ary terms $p_1, \ldots, p_n$ abiding by the equations

$$p_1(x,y,b_1(x,y),\ldots,b_m(x,y)) = x$$
$$p_i(y,x,b_1(x,y),\ldots,b_m(x,y)) = p_{i+1}(x,y,b_1(x,y),\ldots,b_m(x,y))$$
$$p_n(y,x,b_1(x,y),\ldots,b_m(x,y)) = y$$

and, for each $i = 1, \ldots, n$, we have

$$p(0,0,c_1(z),\ldots,c_m(z)) = z$$

The author then considers varieties abiding by the condition (P) *locally*, i.e., varieties in which each fiber $\mathrm{Pt}_{\mathbb{C}}(X)$ of the fibration of points

$$\pi : \mathrm{Pt}(\mathbb{C}) \to \mathbb{C}$$

is pervious to the condition (P). Every pointed variety $\mathbb{C}$ obeying the condition (P) has normal projections in the sense of [*Z. Janelidze*, Theory Appl. Categ. 11, 212–214 (2003; Zbl 1018.18001)], though the converse does not hold. What is therefore remarkable, it turns out (Theorem 3.7) that a variety is pursuant to the condition (P) locally iff it has normal local projections [*Z. Janelidze*, Georgian Math. J. 11, No. 1, 93–98 (2004; Zbl 1052.18007)]. It is also shown that the condition (P) and its local version may be regarded as variants of Gumm's shifting lemma [*H. P. Gumm*, Geometrical methods in congruence modular algebras. Providence, RI: American Mathematical Society (AMS) (1983; Zbl 0547.08006)], meaning that any cogruence modular variety stands by the condition (P) locally.

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MSC:

08B05 Equational logic in varieties of algebras
08B10 Congruence modularity and generalizations in varieties of algebras
18A30 Limits; colimits
18B99 Special categories
03C05 Universal algebra (model theory)

Keywords:

product preserves coequalizers; stability of coequalizers under product; product commutes with coequalizers; normal projections; shifting lemma

Full Text: Link

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