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Handbook of quantum logic and quantum structures. Quantum logic. With a foreword by Anatolij Dvurečenskij. (English) Zbl 1184.81003

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The twenties and thirties of the last century were truely exciting times when it was witnessed that quantum mechanics was emerging. In 1936 *J. von Neumann* and *G. Birkhoff* published a joint paper [Ann. Math. (2) 37, 823–843 (1936; Zbl 0015.14603] on the logical foundation of quantum mechanics, marking the birth of a field of reseach which has become known as quantum logic. The most fundamental problems that quantum mechanics raises are conceptual in nature, and there is no consensus of what the proper interpretation of quantum mechanics should be. It is the mathematical and logical investigation of the various aspects of quantum mechanics that constitutes the topic of the present handbook.

The Birkhoff-von Neumann paper triggered, after some time of dormancy admittedly, a rapid development of quantum logical research. The Birkhoff-von Neumann concept of quantum logic is skillfully reviewed by M. Redèi. In 1968 Putnam published a controversial paper entitled "Is logic empirical?" and in 1971 Greechie and Gudder wrote a similar paper entitled "Is a quantum logic a logic" [Helv. Phys. Acta 44, 238–240 (1971)]. The study in the latter direction is discussed by *M. Pavičić* and *N. D. Megill*, while the former question is revisited by *G. Bacciagaluppi*.

Various schools of thought have emerged since Birkhoff and von Neumann's epoch-making paper. A school of thought was initiated by *C. Piron*'s [Helv. Phys. Acta 37, 439–468 (1964; Zbl 0141.23204)]. The school, which has become known as the Geneva school, aimed at reconstructing the formalism of quantum mechanics from first principles. Diederik Aerts, who was once a student of Piron, wrote a readable exposition on this direction of research. The study of generalized Hilbert spaces, nowadays also called orthomodular spaces, developed into a research subject of its own, and recently *M. P. Solèr* [Commun. Algebra 23, No. 1, 219–243 (1995; Zbl 0827.46019)] proved a groundbreaking theorem that an infinite-dimensional generalized Hilbert space containing an orthonormal base is isomorphic with one of the three standard Hilbert spaces. *A. Prestel* gives a complete proof of Solèr's theorem, which is identical with that he published earlier [Manuscr. Math. 86, No. 2, 225–238 (1995; Zbl 0830.46016)].

The operational statistics approach that was developed in an extensive series of papers by Charles Randall and David Foulis in Amherst is reviewed by Wilce's paper in the handbook. Another operational quantum logic, which has Mackey, Varadarajan, the Geneva School and the Amherst school as its predecessors, is dealt with by David J. Moore and Frank Valckenborgh.

Some sophisticated efforts have resulted in linking the logic of quantum mechanics to mainstream logic. Nishimura's paper in the handbook reviews Gentzen methods in quantum logic, while Maria Luisa Dalla Chiare, Robert Giuntini and others' lengthy (100 pages!) paper reviews the relationship of quantum logic with linear logic, paraconsistent logic and other nonclassical logics. *R. Chadha, P. Mateus* and *A. Sernadas*'s paper presents a new extension of classical logic to quantum systems [cf. *P. Mateus* and *A. Sernadas*, Logics in artificial intelligence. 9th European conference, JELIA 2004, Lisbon, Portugal, September 27–30, 2004. Proceedings. Berlin: Springer. Lecture Notes in Computer Science 3229. Lecture Notes in Artificial Intelligence, 239–251 (2004; Zbl 1111.81304), *P. Mateus, A. Sernadas* and *C. Sernadas*, Essays on the foundations of mathematics and logic. Monza: Polimetrica. Advanced Studies in Mathematics and Logic 1.1, 165–194 (2005; Zbl 1151.03324), *P. Mateus, J. Rasga* and *C. Sernadas*, Log. J. IGPL 13, No. 2, 173–199 (2005; Zbl 1086.03015)].

The advent of quantum information and computation has breathed new life into basic quantum mechanics, asking new kinds of questions on the theory and reawaking new interest in the foundations of quantum mechanics. The article of *S. Abramsky* and *B. Coecke* gives an approach to measurements and classical information based on biproducts, emphasizing the branching structure of measurements due to their probabilistic outcomes. The article assumes a modest familiarity with basic notions of category, including symmetric monoidal categories.

K. Svozil's article deals with contexts in quantum, classical and partition logics. Contexts in the author's usage refers to a maximal collection of co-measurable observables bundled together to form a quasi-classical mini-universe within some larager nonclassical structure. Some of the material has already been published in [*K. Svozil*, Quantum logic. Springer Series in Discrete Mathematics and Theoretical Computer Science. Singapore: Springer (1998; Zbl 0922.03084)].

The editors can not stay silent in this exciting area of research, so that they have written on a holistic logic. The concept reflects two intuitions that inevitably arise in connection with quantum mechanics, namely the intuition of nonmonotonicity as expressed by the uncertainty relations and the intuition of holicity, which pervades the literature on the foundations of quantum mechanics.

P. Bruza, D. Widdows and *J. Woods*' article develops the suggestion that cognition down below has a strcuture strikingly similar to the physical structure of quantum states. This is not an idea that originates with these authors. It is well known that there exist mathematical models from the cognitive science of cognition down below that have certain formal similarties to quantum mechanics. The authors take this idea very seriously herein.

Finally *S. Titani*'s article deals with quantum set theory, using a kind of classical implication as an extrinsic operation. The reader is highly recommended to read [*M. Ozawa*, J. Symb. Log. 72, No. 2, 625–648 (2007; Zbl 1124.03045)] for quantum set theory.

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MSC:

81-00	Reference works (quantum theory)
03-00	Reference works (mathematical logic)
00B15	Collections of articles of miscellaneous specific interest
03G12	Quantum logic
81Pxx	Axiomatics, foundations, philosophy of quantum theory

Keywords:

quantum logic; category theory; nonmonotonic logic; quantum set theory; cognitive science; quantum axiomatics; paraconsistent logic; linear logic; fuzzy logic; operational quantum logic; axiomatic quantum mechanic

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